Computer Graphics
MTAT.03.015

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Study IT in .ee
Standard Graphics Pipeline

Last week:
- Construct geometry
- Define transformations
- Assign material properties

This week:
- Culling & Clipping
- Rasterization
- Fragment Shading
- Visibility Tests
- Blending

Vertex Transformations
- Vertex Shader
  - Object's local space → viewport space
  - Determine front-facing triangles
  - Determine which vertices are visible

Fragment Shader
- Calculate correct color values
- Is the fragment visible?
- Blend together multiple fragments
Points and Vectors

- In computer graphics we distinguish:
  - **Point** – a location in space (location vector, *kohavektor*)
  - **Vector** – a direction in space (direction vector, *suunavektor*)
Points and Vectors

- Both are elements of a 2-, 3- or 4-dimensional vector space over the field $\mathbb{R}$.
- More precisely, elements of a coordinate space.
- So both are vectors in terms of algebra.
- We distinguish them because some operations make sense for vectors, some for points.
- A space that contains both of them and defines an addition between a point and a vector is called an affine space.
- More precisely, an Euclidean space.
Points and Vectors

Vector space of points

Vector space of vectors
Points and Vectors

- Given a vector space over $\mathbb{R}^2$ with a basis and the origin, all the elements of the vector space can be represented as a...
Points and Vectors

\[ \mathbf{v} = \alpha_0 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha_1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

So, the scalar coefficients for our \( \mathbf{v} \) would currently be?
Points and Vectors

- Because the elements of our vector space are $n$-tuples, we can call it a coordinate space.

Coordinate space of points

Coordinate space of vectors
Points and Vectors

• Besides just doing operations between points, or between vectors, we want to do operations between them.

• Or do we? Can you think of an operation we would want to do between a point and a vector?
Points and Vectors

- When we put those two spaces together, we get an affine space.
Points and Vectors

- Is this a point or a vector?
  \[ x = \begin{pmatrix} 3 & 2 \end{pmatrix} \]

- What about this?
  \[ x = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \]
Points and Vectors

- **Row-major** and **column-major** formats.
- Which is which?
- How to get from one to another?

\[
\begin{pmatrix}
3 \\
2
\end{pmatrix} \quad \begin{pmatrix}
3 & 2
\end{pmatrix}
\]

Sometimes the reader is expected to guess which one is used based on the context.

Good explanation:
Points and Vectors

• **Homogeneous coordinates** – a notation where we add an additional coordinate to distinguish between points and vectors.

\[
p = (x \ y \ z) = \left( \frac{x}{z}, \frac{y}{z} \right)
\]

\[
p = (x \ y \ z \ w) = \left( \frac{x}{w}, \frac{y}{w}, \frac{z}{w} \right)
\]
Points and Vectors

- In 2D homogeneous coordinates:
  - Point
    \[ p = (x \ y \ z) \quad z \neq 0 \]
  - Vector
    \[ p = (x \ y \ z) \quad z = 0 \]
  - Vector is a point located in infinity
Points and Vectors

- What should $z$ be if you want to define a point located at $(x, y)$?
- How does addition work now?
- Addition between two vectors?
- Addition between two points?
- Subtraction of points?
Line

- You probably know about the **implicit** line equation: \( y = a \cdot x + b \)
  
  It defines the relationship between the coordinates.

- Can also be used to test if a given point is on the line or not. 
  How?
Line

- How can we represent a line in our affine space?
- We do not know $a$ (the slope) or $b$ (y-intercept).
- What would we need to know to represent a line?
Line

- Two points that the line passes
- One point and a direction vector
Line

\[ line = (1 - \alpha) \cdot A + \alpha \cdot B \quad \text{line} = A + \alpha \cdot d \]

\[ d = B - A \]
Line Segment

- Knowing that: \( \text{line} = (1 - \alpha) \cdot A + \alpha \cdot B \)

How to represent a line segment?
Triangle

- How about a triangle?
Barycentric Coordinates

- The coefficients of a **convex combination** of the vertices are the **Barycentric coordinates** of all the points inside the triangle.

\[
\text{triangle} = \alpha_0 \cdot A + \alpha_1 \cdot B + \alpha_2 \cdot C
\]

\[
\alpha_i \geq 0, \quad \alpha_0 + \alpha_1 + \alpha_2 = 1
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Barycentric Coordinates

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\]

What are the coordinates of the vertices in the Barycentric system?

Find them for other easy points.
Dot Product

• Useful operation between vectors. Why?

• Definition

  • Geometric: \( u \cdot v = \|u\| \cdot \|v\| \cdot \cos(\angle uv) \)

  • Algebraic: \( u \cdot v = u_0 \cdot v_0 + u_1 \cdot v_1 + u_2 \cdot v_2 \)

• Also called: scalar product, inner product

• \textit{Skalaarkorrutis}
Scalar Projection

• Dot product can be used to project one vector onto another.

• Scalar projection of $u$ onto $v$ is:

$$c = u \cdot \frac{v}{\|v\|} = u \cdot \hat{v}$$

• It gives you the length, how much $\hat{v}$ you have to take in order to reach the orthogonal projection point of $u$.

Eg The Gram–Schmidt process
Cross Product

- Returns a vector orthogonal to the operands.

- Definition
  \[ u \times v = n \cdot \|u\| \cdot \|u\| \cdot \sin(\angle uv) \]
  \[ u \times v = \left( \begin{array}{c} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{array} \right) \]

- Geometric
- Algebraic

- Also called: vector product

- Vektorkorrutis

Direction of the result depends on the handedness of the coordinate system.
Scalar Triple Product

- Definition: \( \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) \)

- Useful in solving a system of equations of vectors, because:

  \[
  \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} u_0 & u_1 & u_2 \\ v_0 & v_1 & v_2 \\ w_0 & w_1 & w_2 \end{vmatrix}
  \]

- We can see this in Basic II, with triangle-ray intersection testing.

- *Segakorrutis.*
What was important for you today?

What more would you like to know?

Next time: Transformations
(scale, shear, rotate, translate)