Computer Graphics

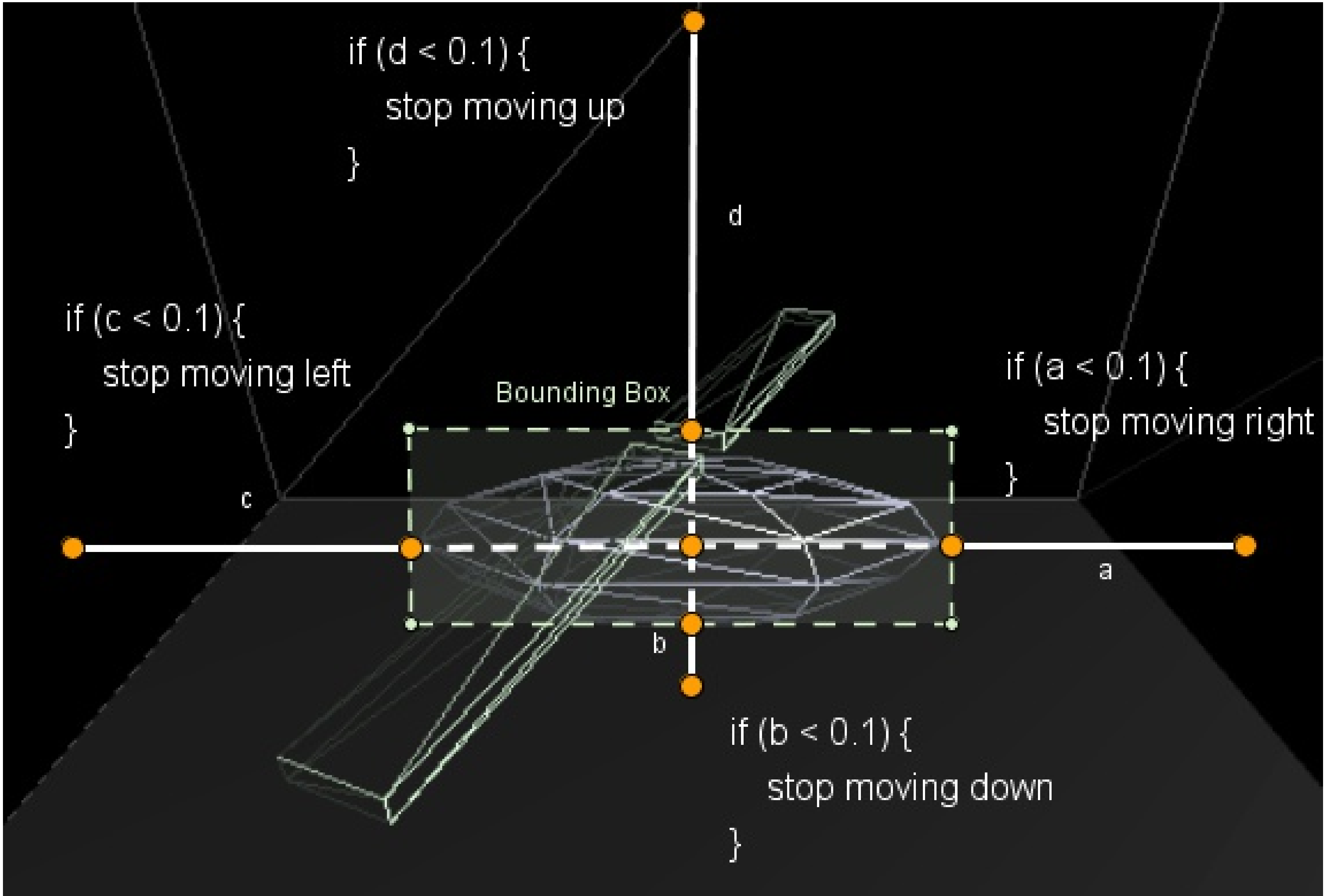
MTAT.03.015

Raimond Tunnel

TARTU ÜLIKOOL · UNIVERSITAS TARTUENSIS · 1632

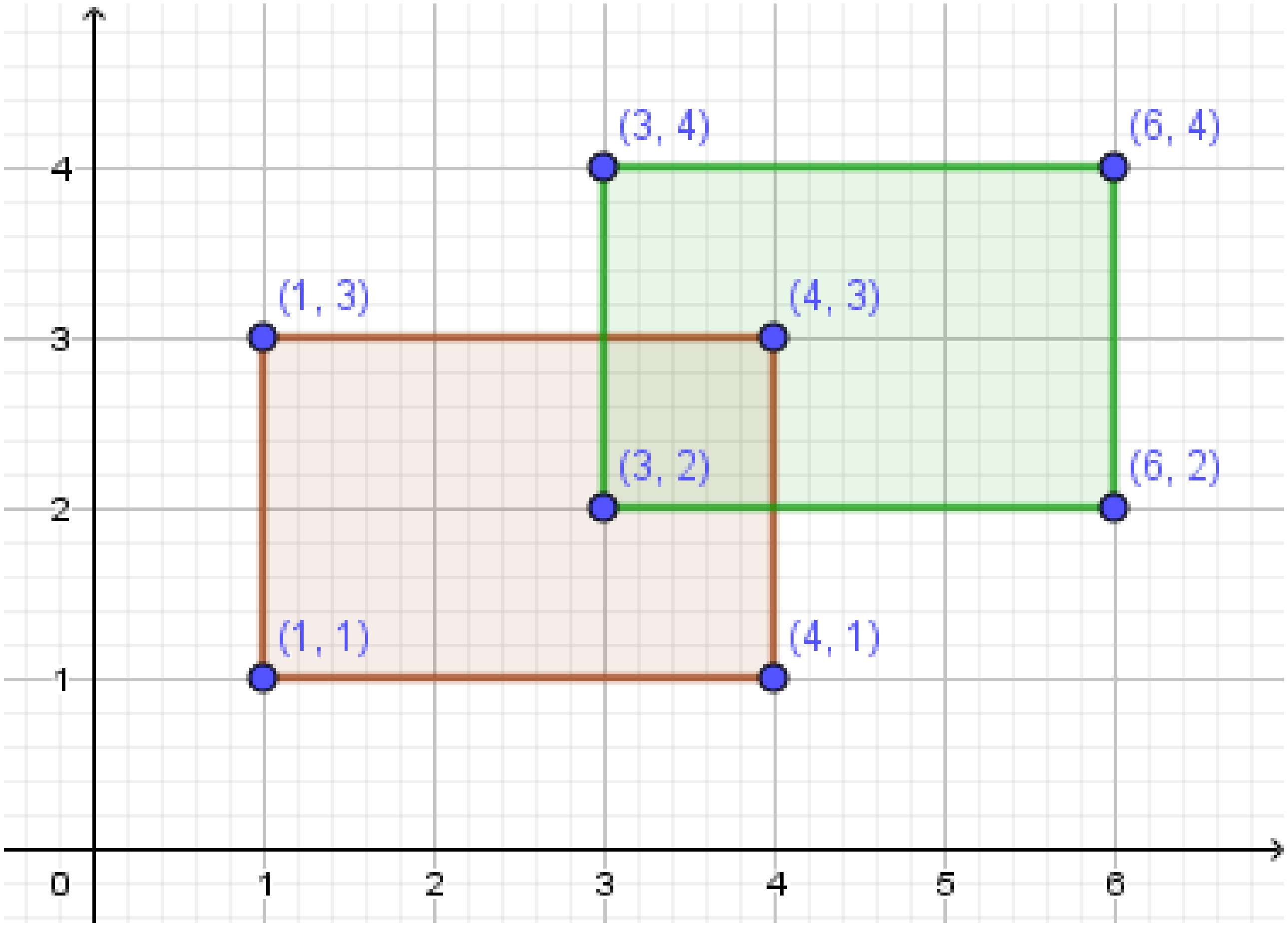
Study IT in .ee

The Road So Far...



Bounding Box

- With bounding boxes you can detect collisions between boxes.



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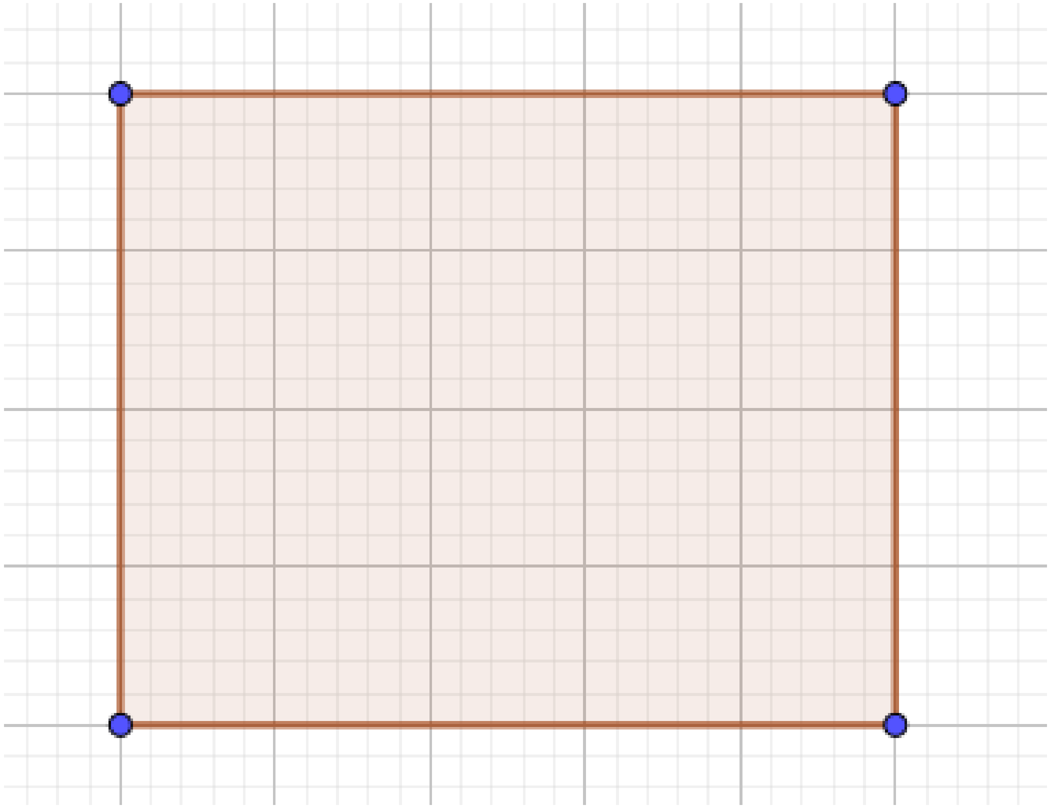
- Our hangar just happens to be a box.



Bounding Box

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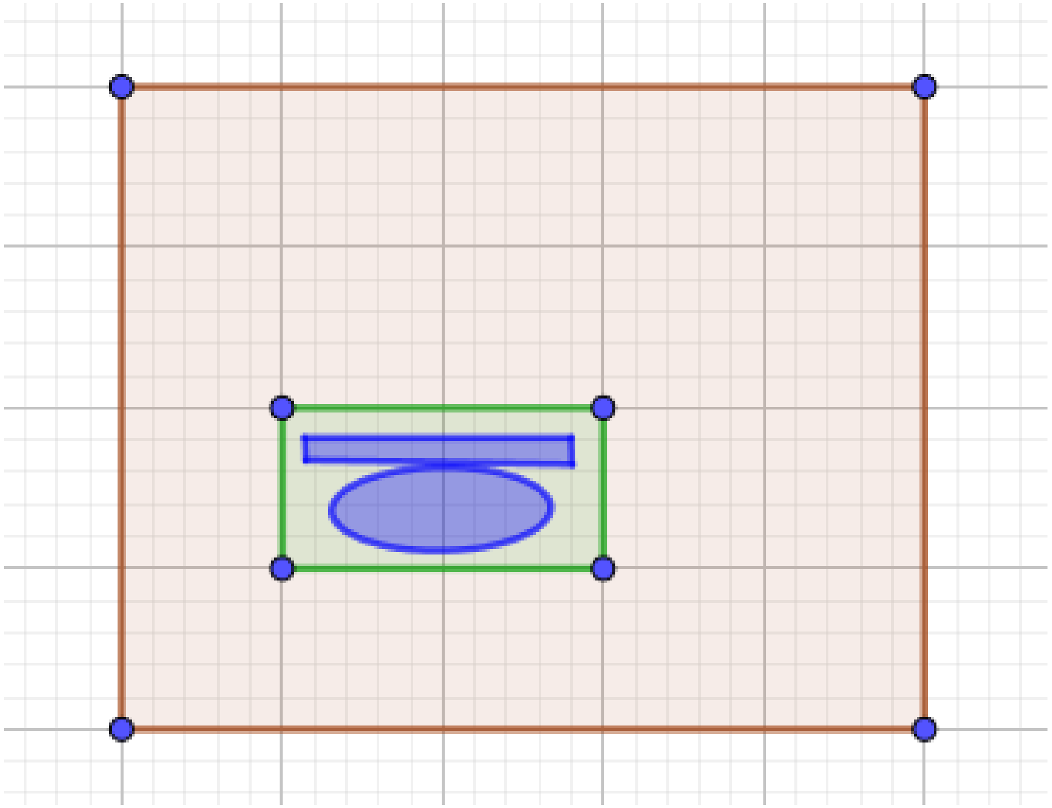
- The chopper is not a box, but the collision approximation with a bounding box seems ok.



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- Our hangar just happens to be a box.
- The chopper is not a box, but the collision approximation with a bounding box seems ok.
- The bounding box is **axis-aligned**.

Bounding Box

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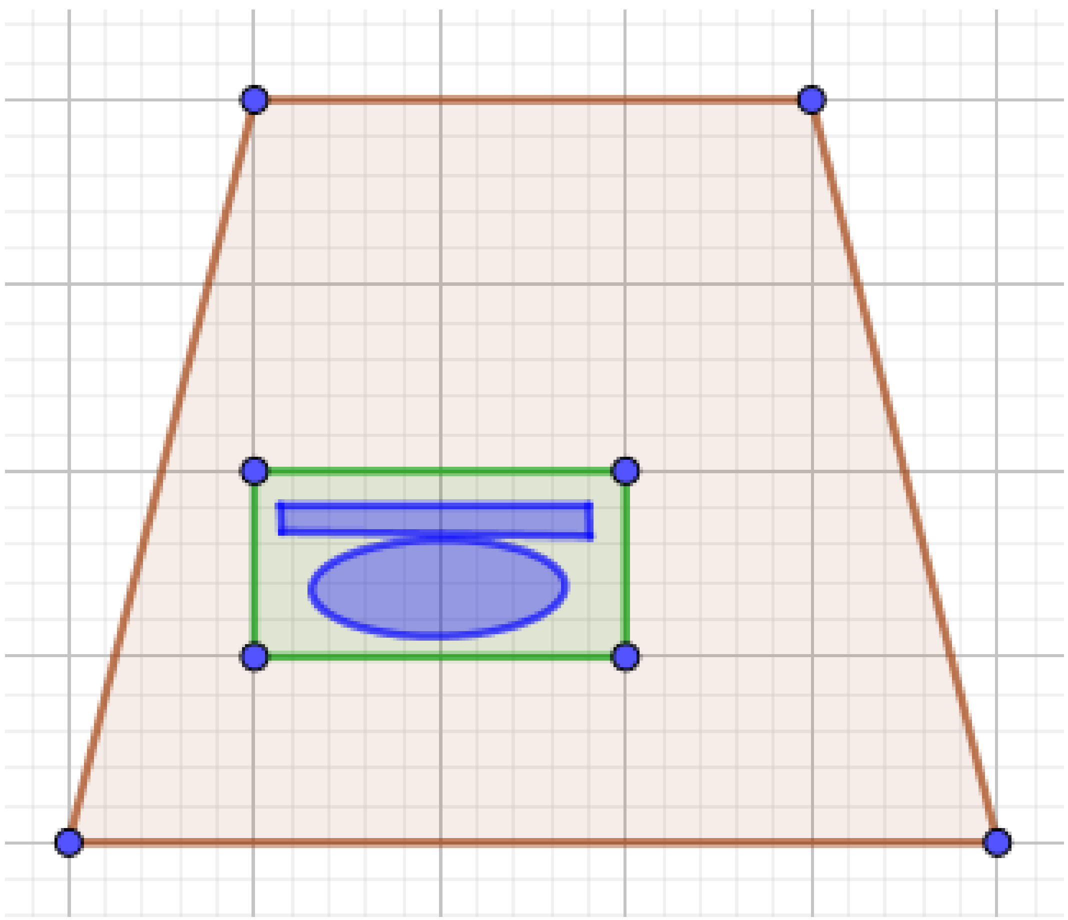
- Our hangar just happens to be a box.
- The chopper is not a box, but the collision approximation with a bounding box seems ok.
- The bounding box is **axis-aligned**.
- Some of you wrote 4 if-statements.
  That is a (kind of) bounding box collision detection for those specific boxes (around chopper, the hangar).

Collision Detection

- What if the hangar walls were rotated? Can not assume that all walls are always axis-aligned..



Collision Detection

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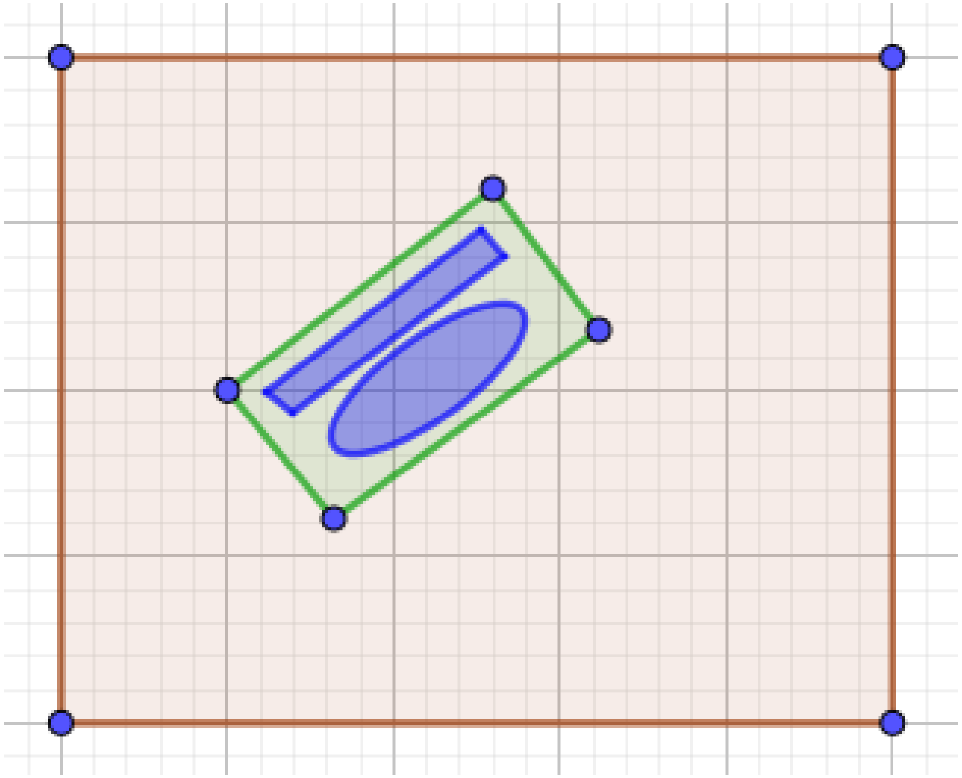
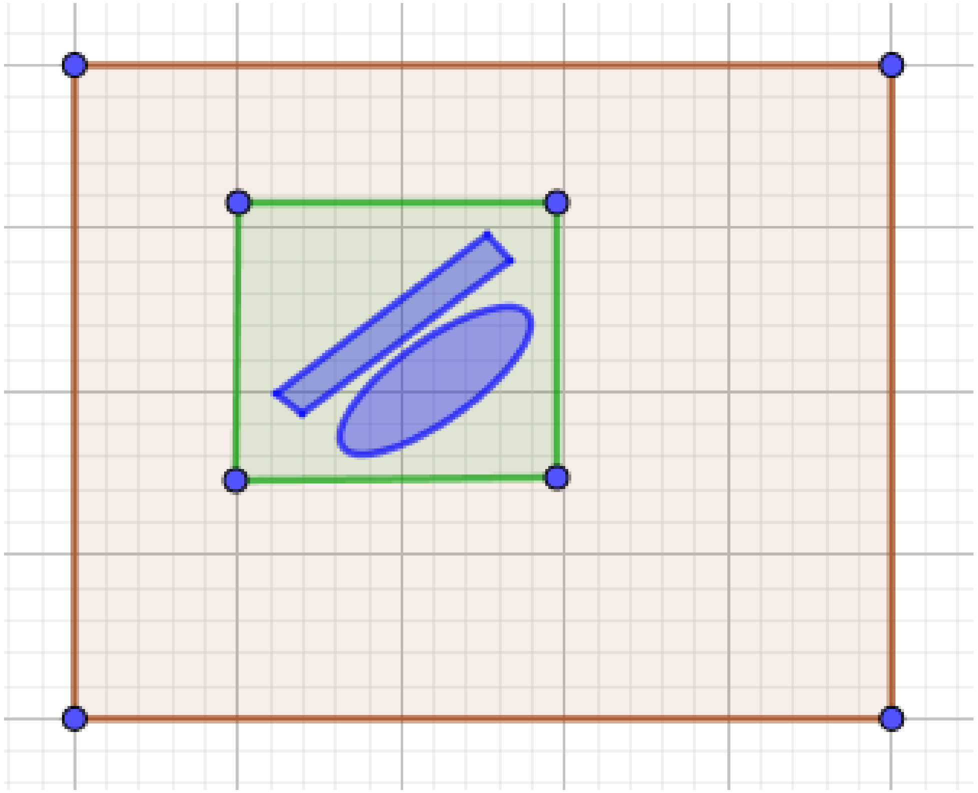
- What if the chopper was rotated?





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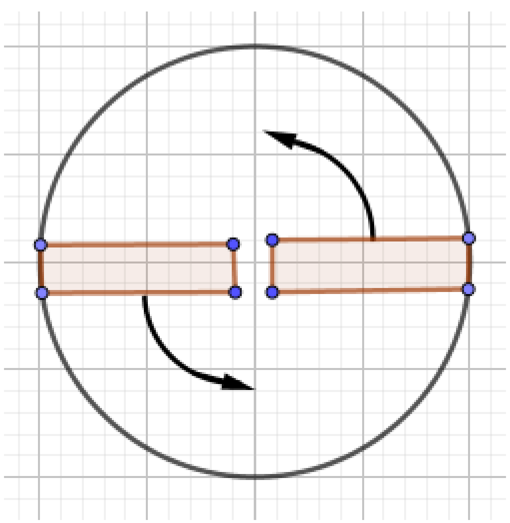
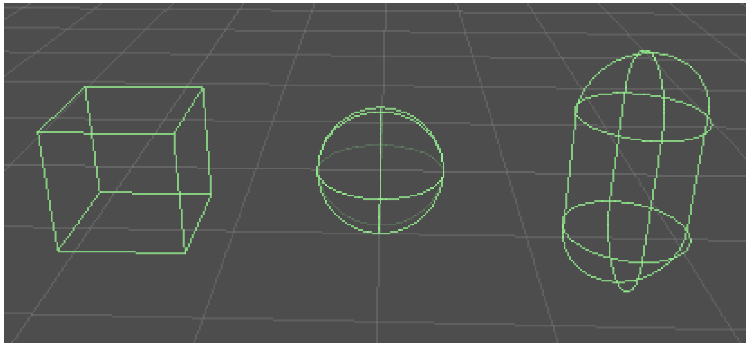
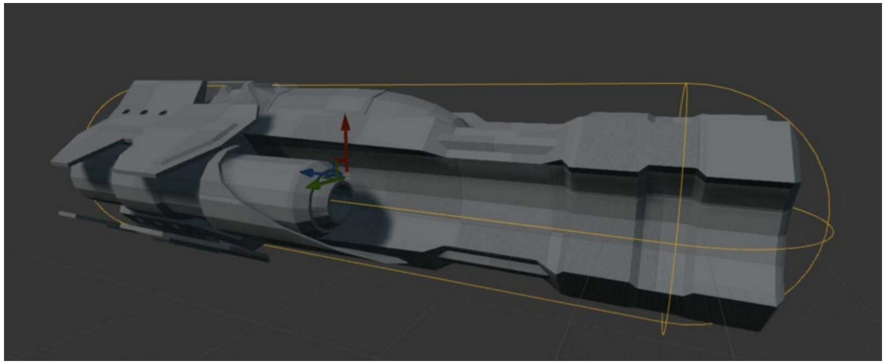
- What if the chopper rotated?
- The rotating blades actually would need a cylinder to minimally bound them.



Collision Detection

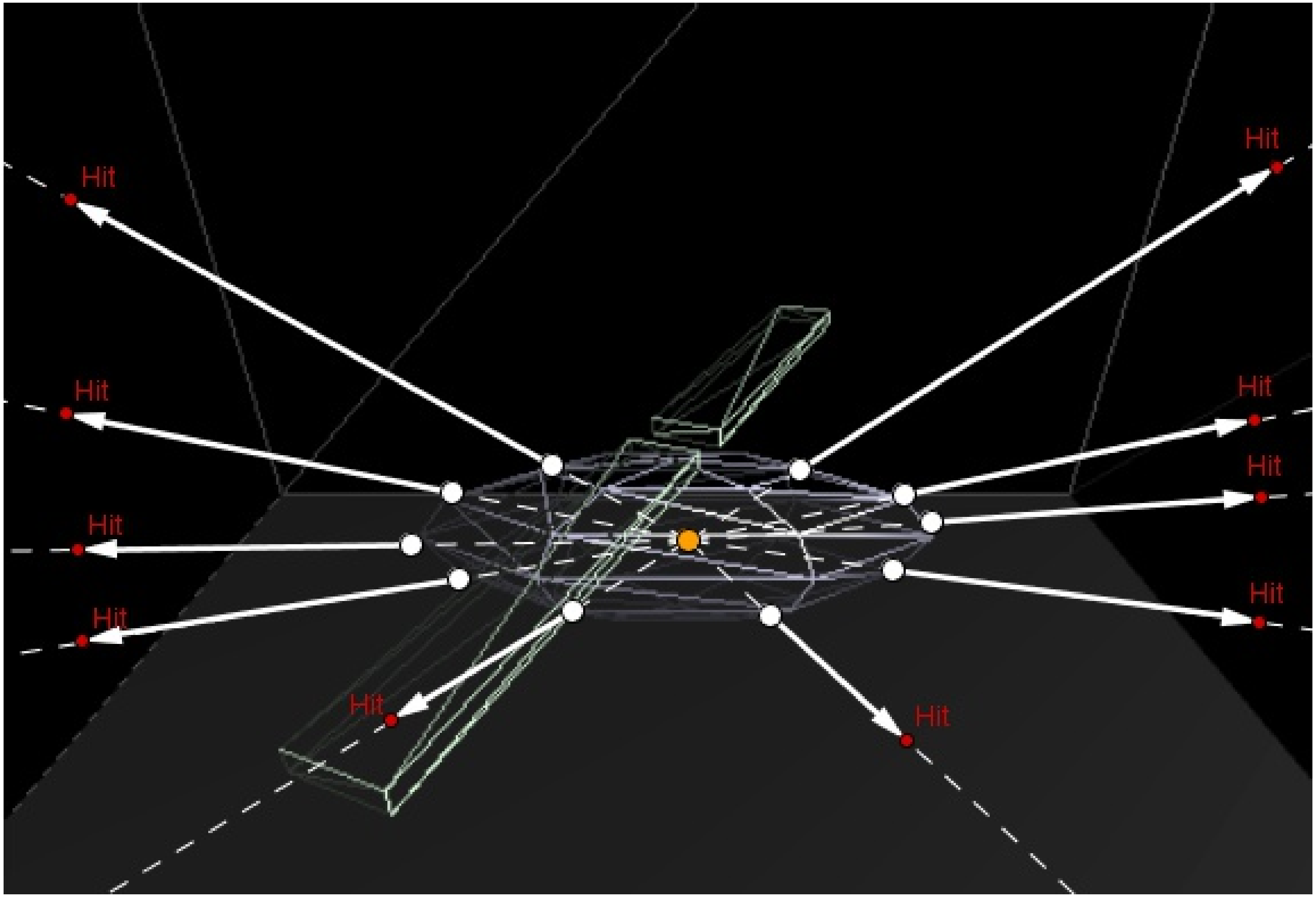
- What if the hangar walls were rotated? Can not assume that all walls are always axis-aligned.
- What if the chopper rotated?
- The rotating blades actually would need a cylinder to minimally bound them.
- Bounding objects provide a fast and rough approximation.





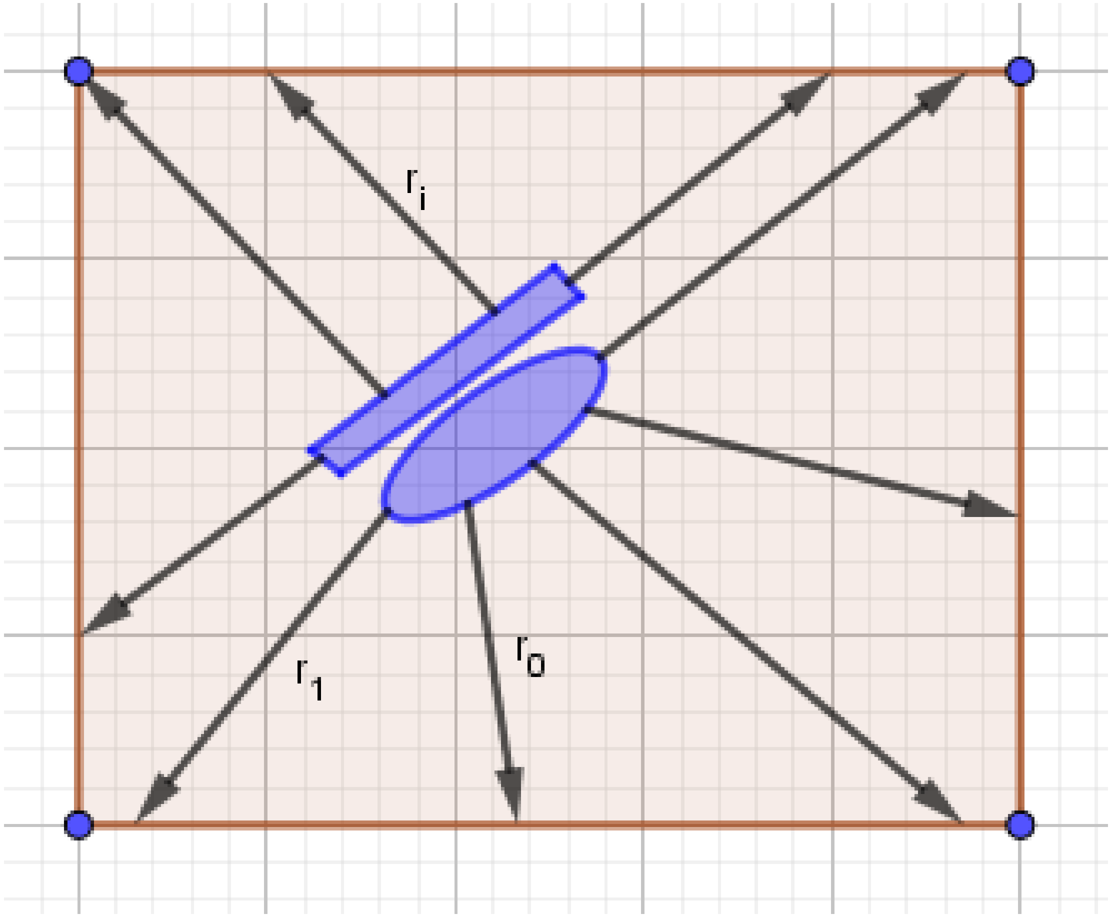
Ray Casting

- Cast rays out of some vertices, following the vertex normal.



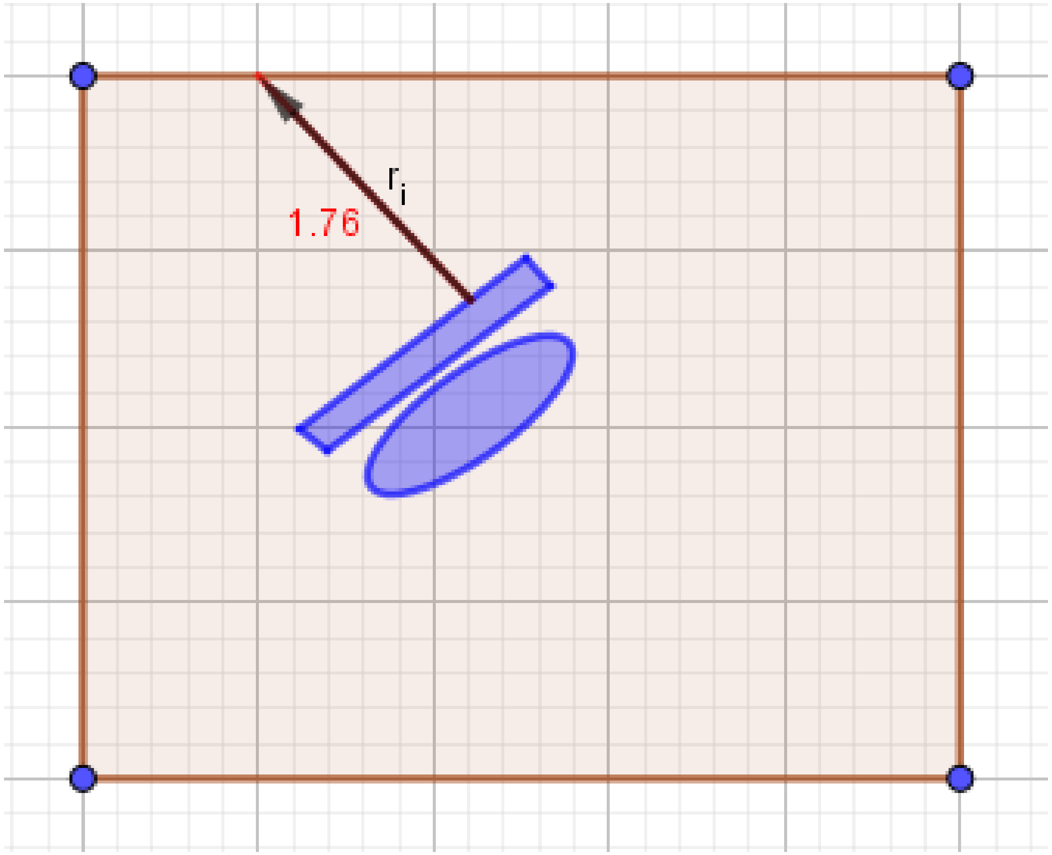
Ray Casting

- Detect the first hit of ray and scene geometry.



Ray Casting

- Detect the first hit of ray and scene geometry.
- Measure the distance from the vertex to the hit.



Ray Casting

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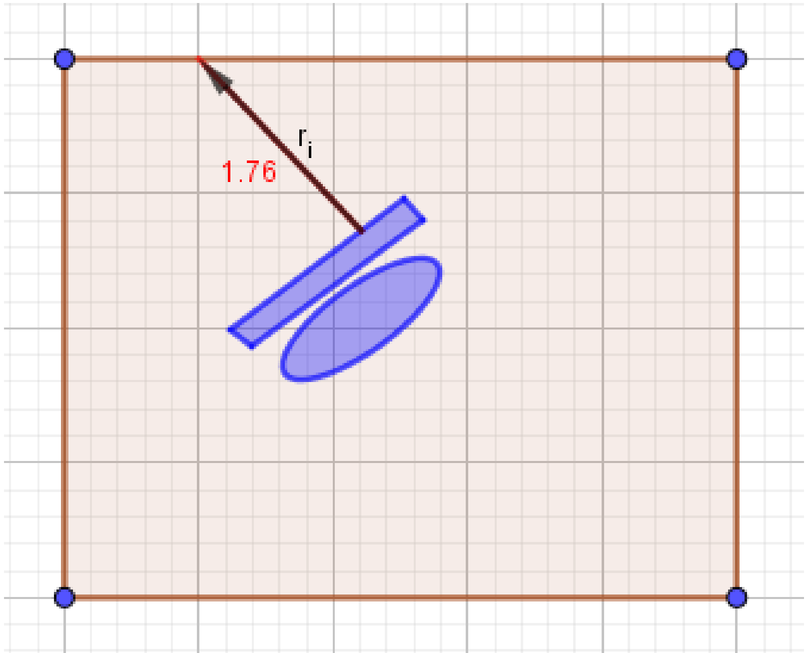
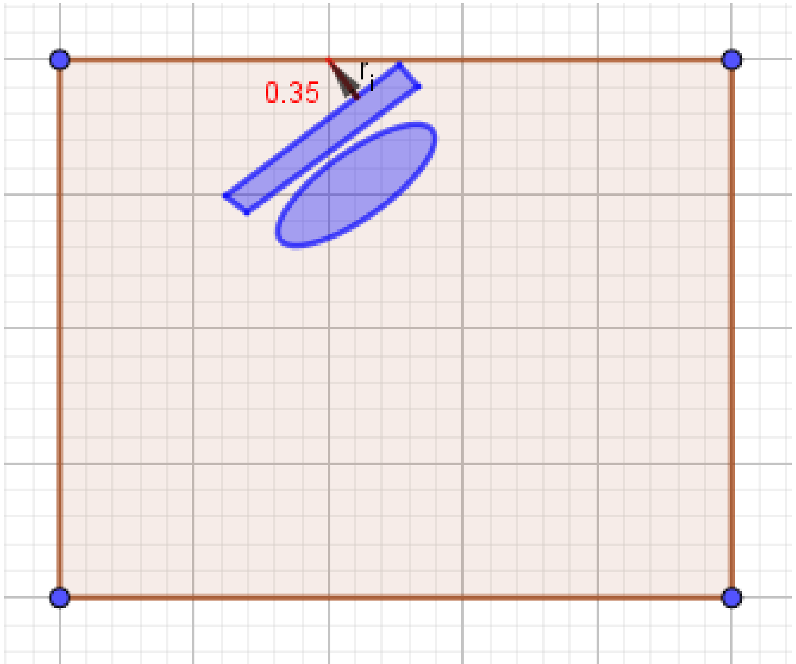
- Measure the distance from the vertex to the hit.
- If the distance is too small, change the chopper's position, speed, acceleation, in order to avoid a collision.



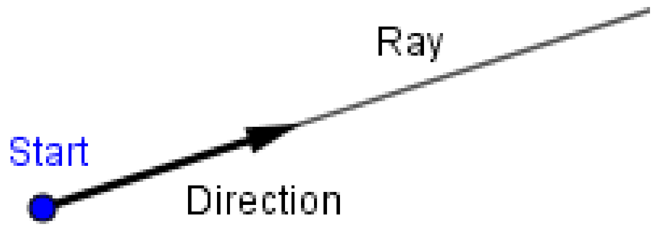


Ray Casting

- Detect the first hit of ray and scene geometry.
- Measure the distance from the vertex to the hit.
- If the distance is too small, change the chopper's position, speed, acceleation, in order to avoid a collision.
- Intersection testing:
  - Intersection testing between a variety of objects: http://www.realtimerendering.com/intersections.html

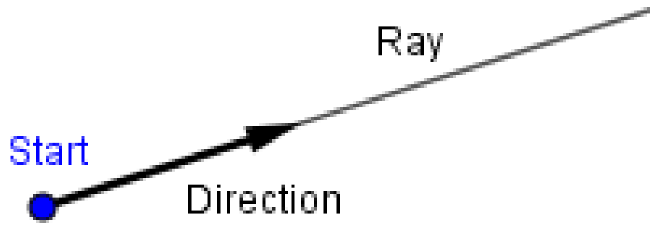
Möller-Trumbore Ray Triangle

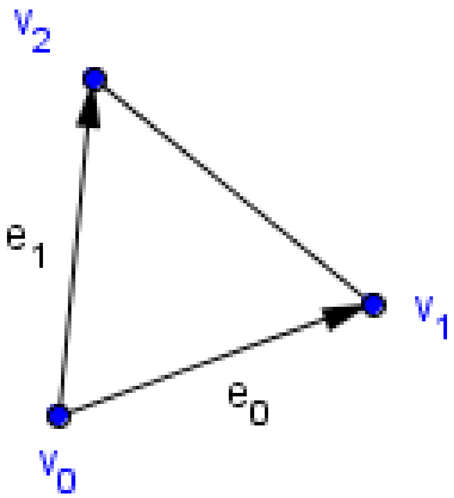
$$Ray(t) = Start + t \cdot Direction$$



Möller-Trumbore Ray Triangle

- $Ray(t) = Start + t \cdot Direction$
- $Triangle(u, v) = v_0 + u \cdot e_0 + v \cdot e_1$





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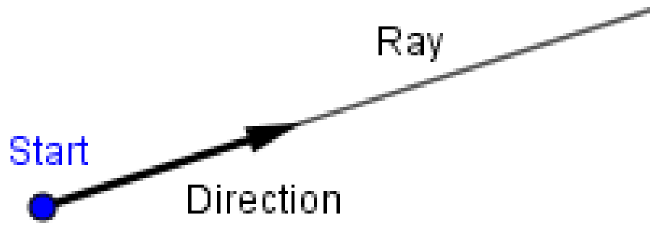
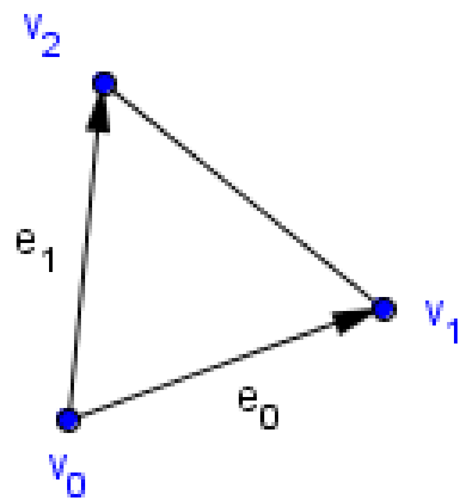
- The u and v are actually Barycentric coordinates of vertices v_1 and v_2.





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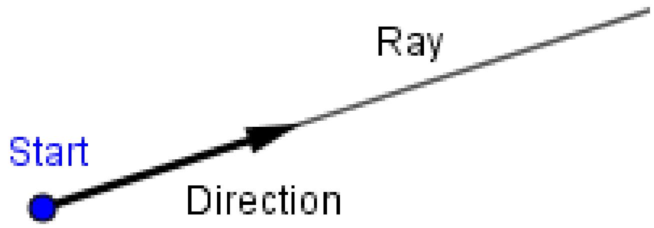

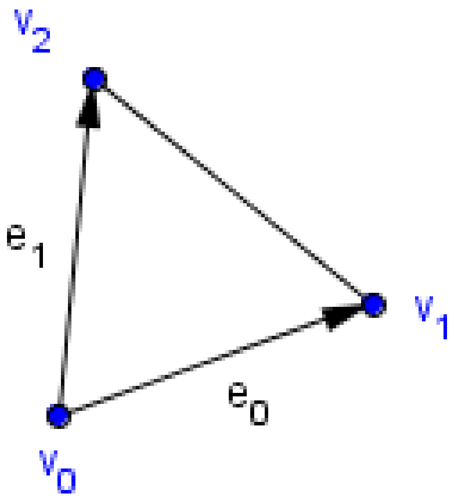

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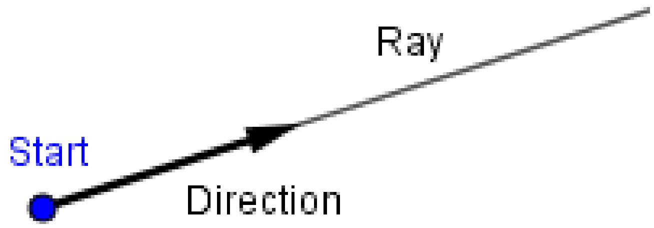
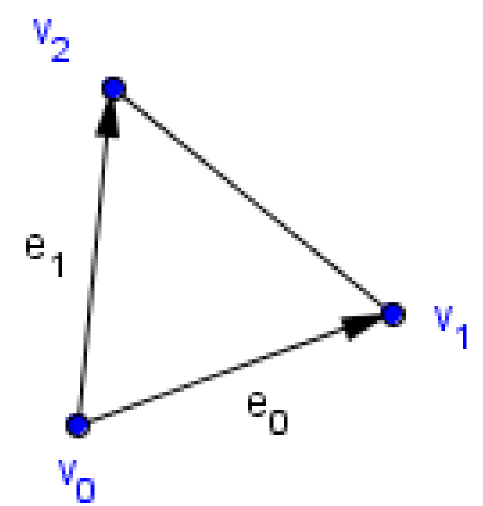
- **Goal is to find a solution to the following equation:**

$$Ray(t) = Triangle(u, v)$$





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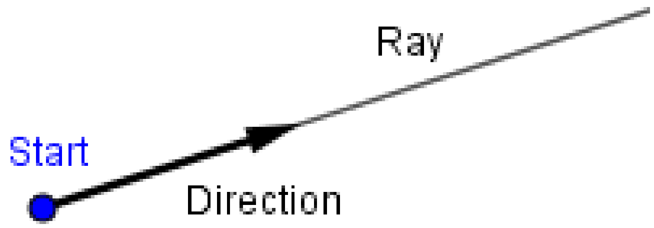

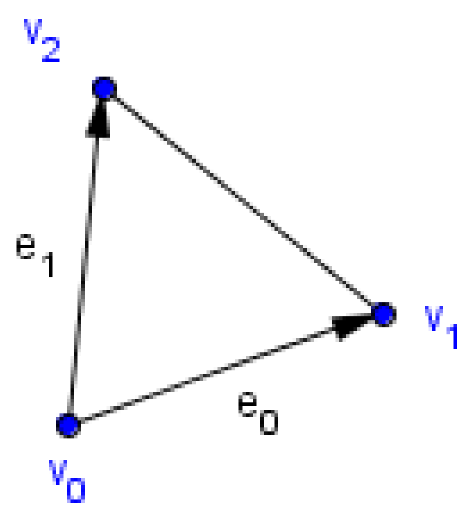

$$Ray(t) = Start + t \cdot Direction = v_0 + u \cdot e_0 + v \cdot e_1 = Triangle(u, v)$$

Möller-Trumbore Ray Triangle

- Let us call S the $Start$ and D the $Direction$.

Möller-Trumbore Ray Triangle

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- We can rearrange the terms to see better.

$$S + t \cdot D = (1 - u - v) v_0 + u \cdot v_1 + v \cdot v_2$$

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$$S-v_0=u\cdot(v_1-v_0)+v\cdot(v_2-v_0)-t\cdot D$$

Moving the constant values to one side...

Parameteres to the other side...

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Notice the triangle basis vectors again...

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Parameteres to the other side...

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$$S - v_0 = u \cdot (v_1 - v_0) + v \cdot (v_2 - v_0) - t \cdot D$$

$$\left((v_1 - v_0) \quad (v_2 - v_0) \quad -D \right) \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = S - v_0$$

Converting into vector form

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$$\left((v_1-v_0) \quad (v_2-v_0) \quad -D\right)\cdot\begin{pmatrix} u \\ v \\ t \end{pmatrix}=S-v_0$$

- We are looking for the unknown vector $\begin{pmatrix} u \\ v \\ t \end{pmatrix}$

Möller-Trumbore Ray Triangle

- We are in 3D, so we have 3 equations for each dimension.

$$\begin{pmatrix} e_0 & e_1 & -D \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = S - v_0$$

Using the basis vectors
for simpler writeup

Möller-Trumbore Ray Triangle

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$$\begin{pmatrix} e_0 & e_1 & -D \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = S - v_0$$

- Cramer's rule

$$x = \frac{|A_x|}{|A|} \quad y = \frac{|A_y|}{|A|} \quad z = \frac{|A_z|}{|A|}$$

$$a_{0,0} \cdot x + a_{0,1} \cdot y + a_{0,2} \cdot z = b_0$$
$$a_{1,0} \cdot x + a_{1,1} \cdot y + a_{2,2} \cdot z = b_1$$
$$a_{2,0} \cdot x + a_{2,1} \cdot y + a_{2,2} \cdot z = b_2$$

A_x - first column replaced by b

A_y - second column replaced by b

A_z - third column replaced by b

Möller-Trumbore Ray Triangle

- With Cramer's rule

$$\begin{pmatrix} e_0 & e_1 & -D \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = S - v_0$$

$$u = \frac{\begin{vmatrix} S_x - v_{0\mathrm{x}} & e_{1\mathrm{x}} & -D_x \\ S_y - v_{0\mathrm{y}} & e_{1\mathrm{y}} & -D_y \\ S_z - v_{0\mathrm{z}} & e_{1\mathrm{z}} & -D_z \end{vmatrix}}{\begin{vmatrix} e_{0\mathrm{x}} & e_{1\mathrm{x}} & -D_x \\ e_{0\mathrm{y}} & e_{1\mathrm{y}} & -D_y \\ e_{0\mathrm{z}} & e_{1\mathrm{z}} & -D_z \end{vmatrix}}$$

etc

Möller-Trumbore Ray Triangle

- With Cramer's rule

$$\begin{pmatrix} e_0 & e_1 & -D \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = S - v_0$$

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- Denote columns

$$b = S - v_0$$

$$u = \frac{\begin{vmatrix} b & e_1 & -D \end{vmatrix}}{\begin{vmatrix} e_0 & e_1 & -D \end{vmatrix}} \quad v = \frac{\begin{vmatrix} e_0 & b & -D \end{vmatrix}}{\begin{vmatrix} e_0 & e_1 & -D \end{vmatrix}} \quad t = \frac{\begin{vmatrix} e_0 & e_1 & b \end{vmatrix}}{\begin{vmatrix} e_0 & e_1 & -D \end{vmatrix}}$$

How to find those determinants?

Möller-Trumbore Ray Triangle

- Scalar triple product: $a \cdot (b \times c) = \begin{vmatrix} a & b & c \end{vmatrix}$

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- Anticommutativity of the cross product:

$$u = \frac{b \cdot (\boldsymbol{D} \times \boldsymbol{e_1})}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})} \qquad v = \frac{e_0 \cdot (D \times b)}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})} \qquad t = \frac{e_0 \cdot (e_1 \times b)}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})}$$

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- Circular shift invariance of scalar triple product

$$v=\frac{D\cdot(\boldsymbol{b}\times\boldsymbol{e_0})}{e_0\cdot(D\times e_1)}\qquad t=\frac{e_1\cdot(\boldsymbol{b}\times\boldsymbol{e_0})}{e_0\cdot(D\times e_1)}$$

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Think about the matrix
elementary row operations...

Möller-Trumbore Ray Triangle

- We can calculate only two cross products

$$u=\frac{b\cdot(\boldsymbol{D}\times\boldsymbol{e_1})}{e_0\cdot(\boldsymbol{D}\times\boldsymbol{e_1})} \qquad v=\frac{D\cdot(\boldsymbol{b}\times\boldsymbol{e_0})}{e_0\cdot(\boldsymbol{D}\times\boldsymbol{e_1})} \qquad t=\frac{e_1\cdot(\boldsymbol{b}\times\boldsymbol{e_0})}{e_0\cdot(\boldsymbol{D}\times\boldsymbol{e_1})}$$

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$$u=\frac{b\cdot P}{\hat{P}} \qquad v=\frac{D\cdot Q}{\hat{P}} \qquad t=\frac{e_1\cdot Q}{\hat{P}}$$

$$Q=(b\times e_0)$$

$$P=(D\times e_1)$$

$$\hat{P}=e_0\cdot P$$

Möller-Trumbore Ray Triangle

- We can calculate only two cross products

$$u = \frac{b \cdot (\boldsymbol{D} \times \boldsymbol{e_1})}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})} \qquad v = \frac{D \cdot (\boldsymbol{b} \times \boldsymbol{e_0})}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})} \qquad t = \frac{e_1 \cdot (\boldsymbol{b} \times \boldsymbol{e_0})}{e_0 \cdot (\boldsymbol{D} \times \boldsymbol{e_1})}$$

$$u = \frac{b \cdot P}{\hat{P}} \qquad v = \frac{D \cdot Q}{\hat{P}} \qquad t = \frac{e_1 \cdot Q}{\hat{P}}$$

$$Q = (b \times e_0)$$
$$P = (D \times e_1)$$
$$\hat{P} = e_0 \cdot P$$

- What happens if: $\hat{P} = e_0 \cdot (D \times e_1) \sim 0$

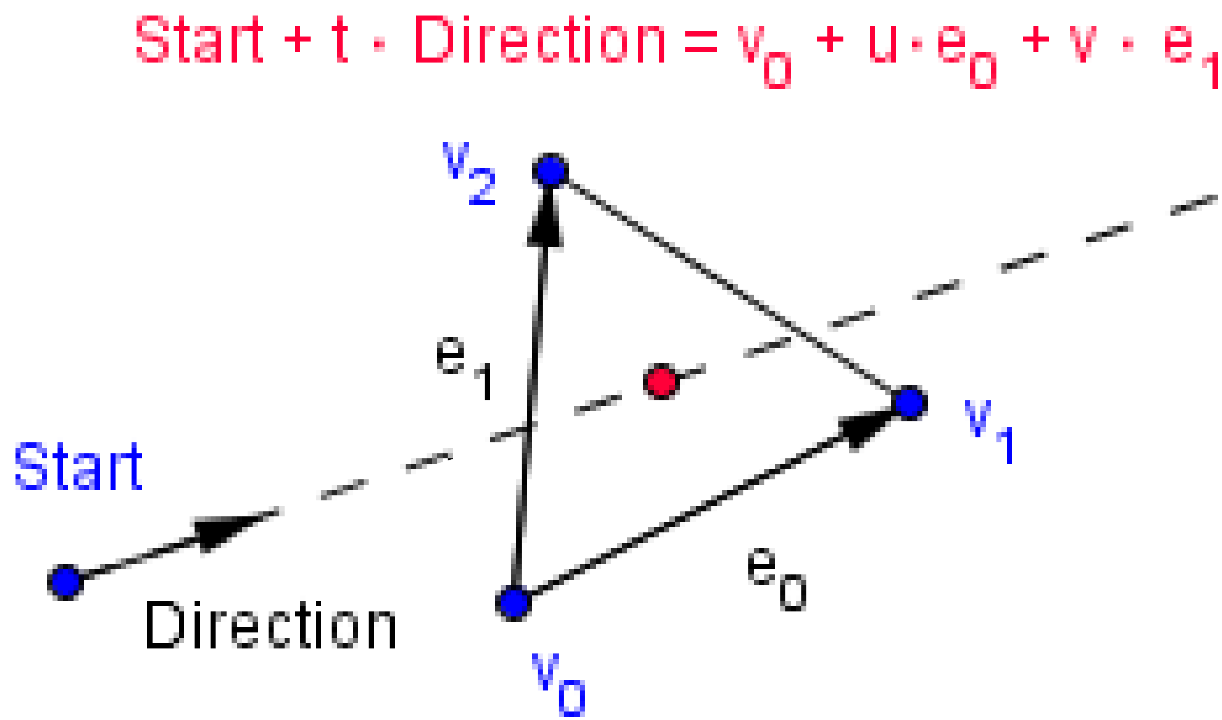
$\hat{P} = e_0 \cdot (D \times e_1) < 0$ $\qquad$ $\hat{P} = e_0 \cdot (D \times e_1) > 0$

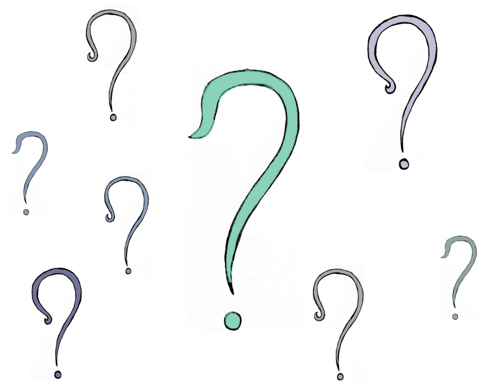
Circular shift can help to visualize this better...

Möller-Trumbore Ray Triangle

- Can it happen, and what does it mean?

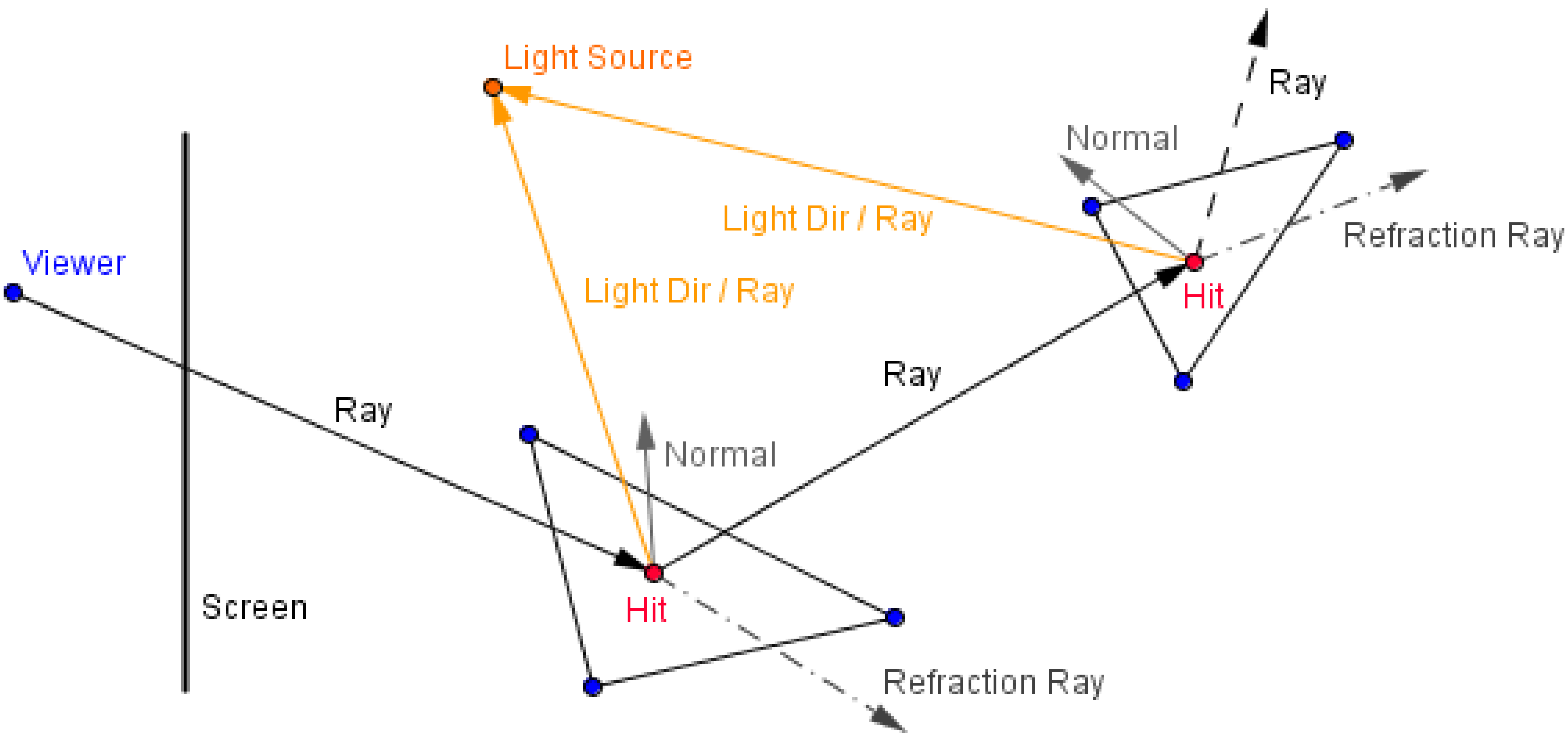
$u<0 \quad u>1 \quad v<0 \quad v>1 \quad u+v>1 \quad t\leq 0$





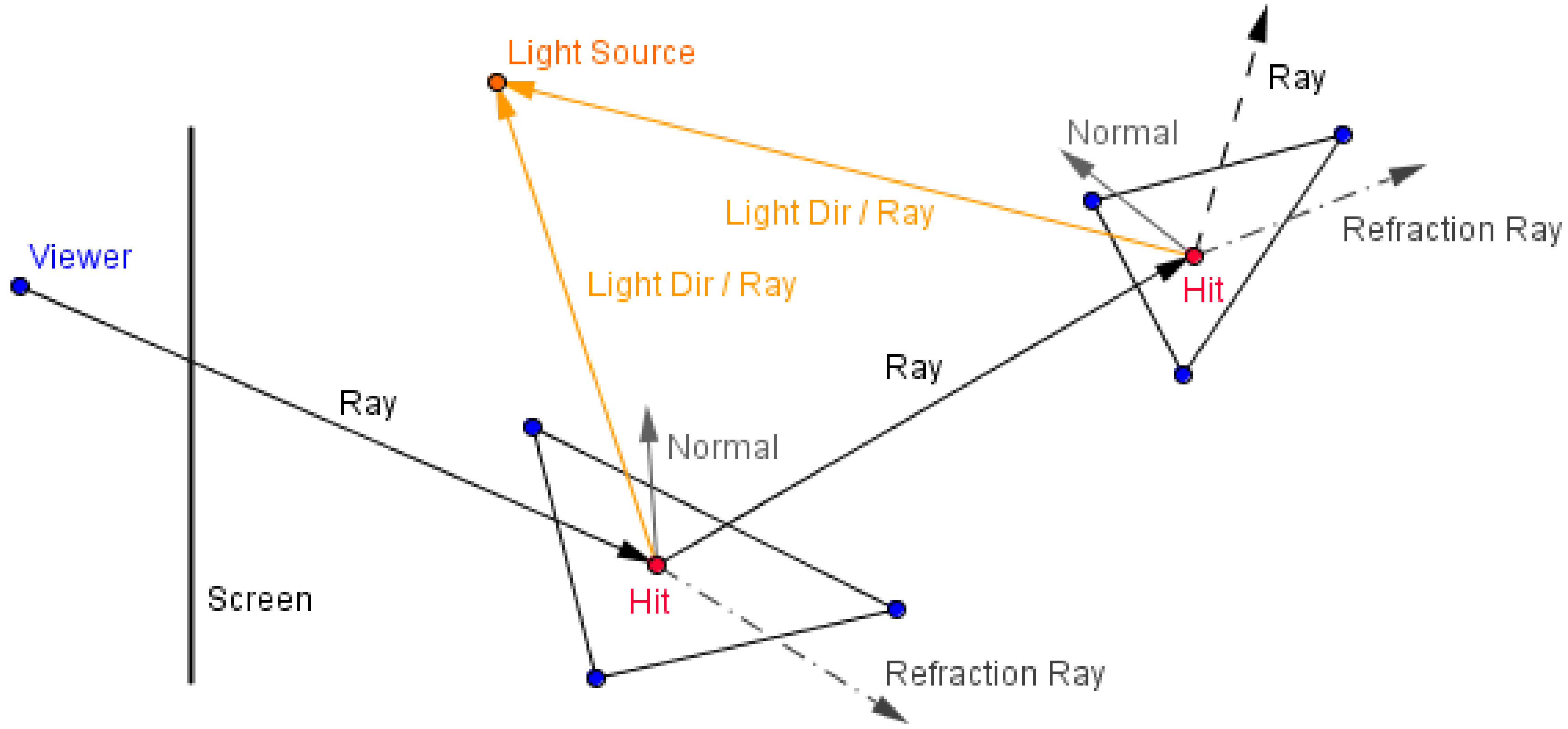
Ray Trace Rendering

- We can use ray tracing to model the light paths (in reverse)



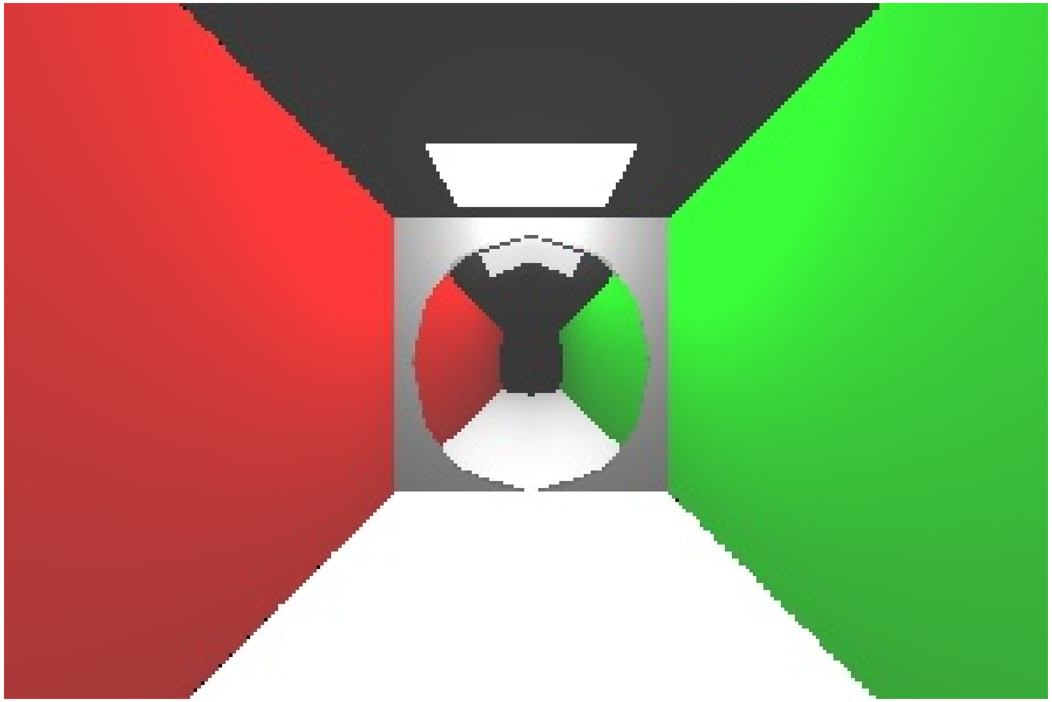
Ray Trace Rendering

- What is the origin of a ray?
- What about the direction?



Ray Trace Rendering

- Accurate way to model reflective / refractive surfaces.



Ray Trace Rendering

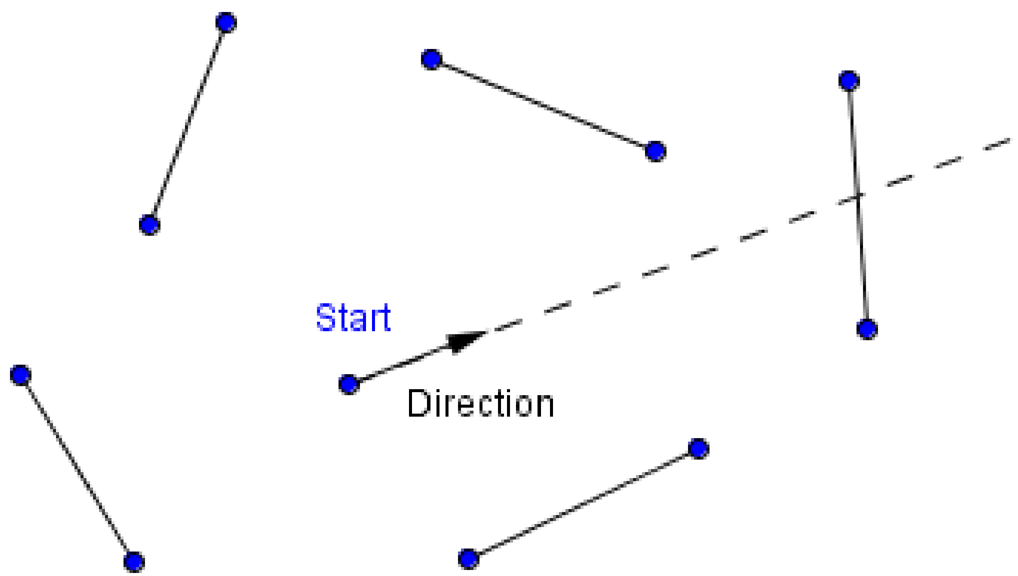
- Accurate way to model reflective / refractive surfaces.
- Quite expensive, we need to test each ray against our geometry.

$$\underbrace{\underbrace{800\cdot 600}_{\text{screen width, height}}\cdot\underbrace{3}_{\text{bounces}}}_{\text{number of rays}}\cdot\underbrace{700}_{\text{triangles (quite few)}}=1008000000$$

That is over a billion intersection tests each frame!

Space Partitioning

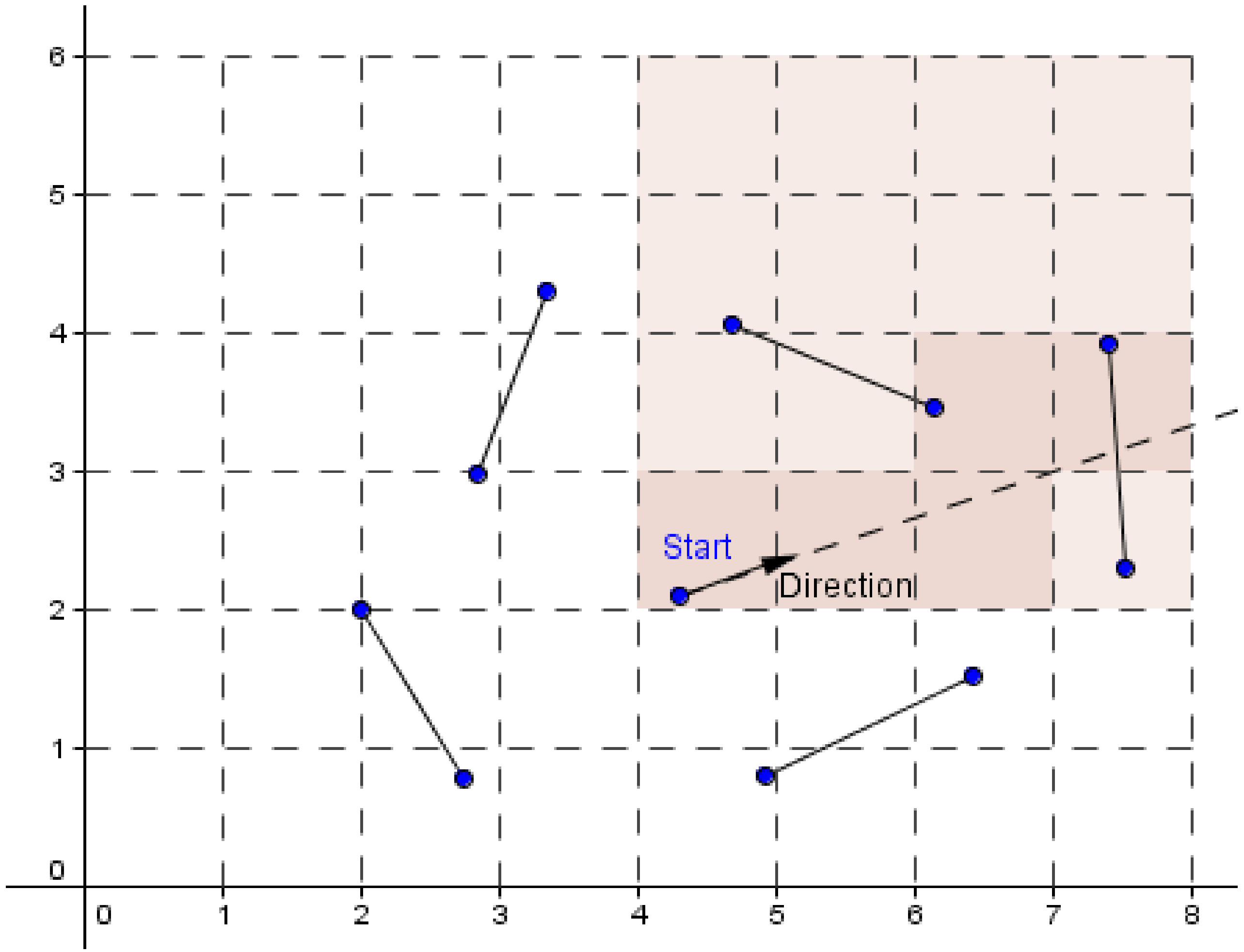
- We can keep our objects in a structure, that lessens the number of intersections we need to test.
- Imagine in 2D a ray and a some line segments.



Any ideas, how to lessen the number of tests?

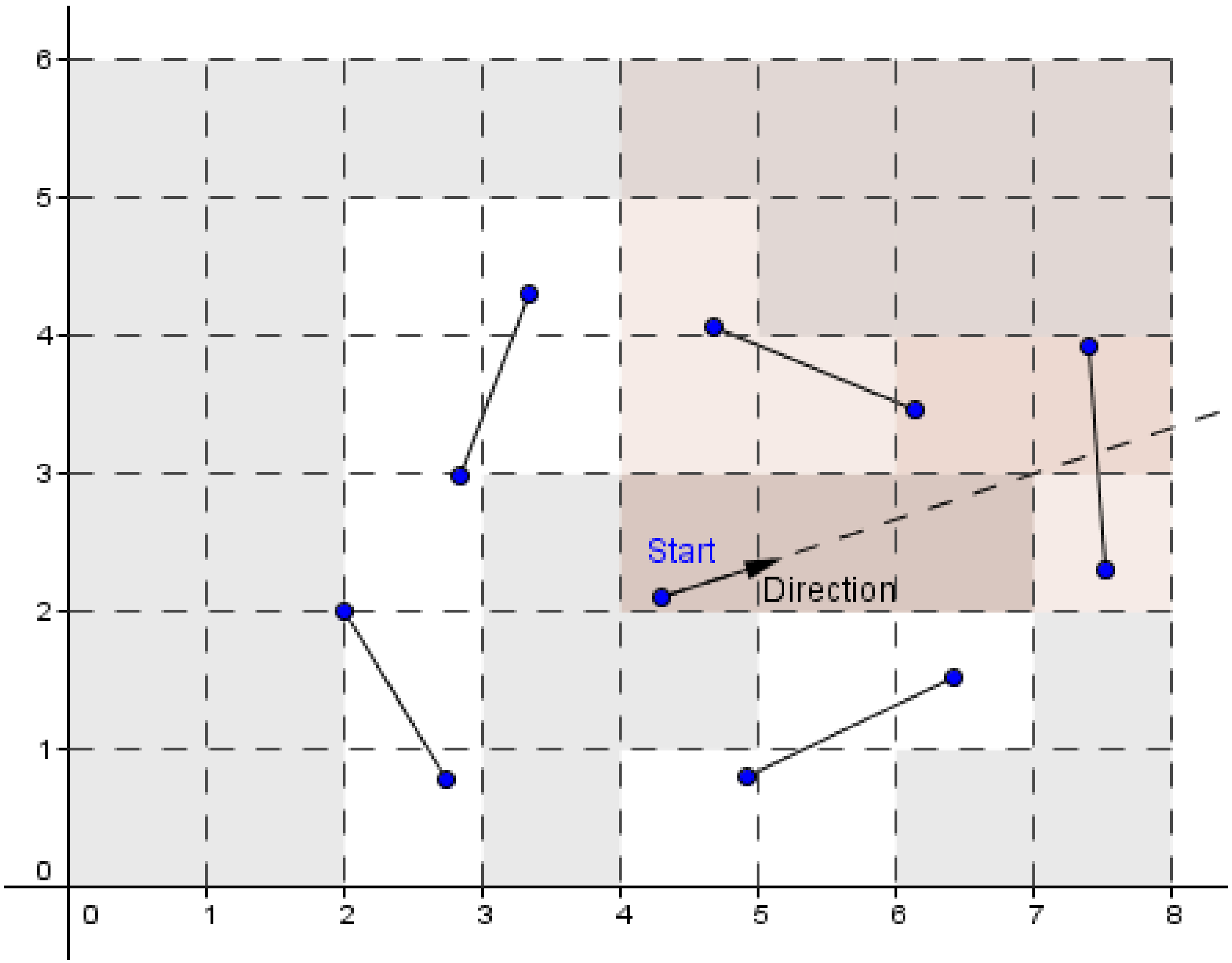
First Idea: Axis-Aligned Grid

- We can limit the number of grid cells to check, by accounting for the ray's direction.



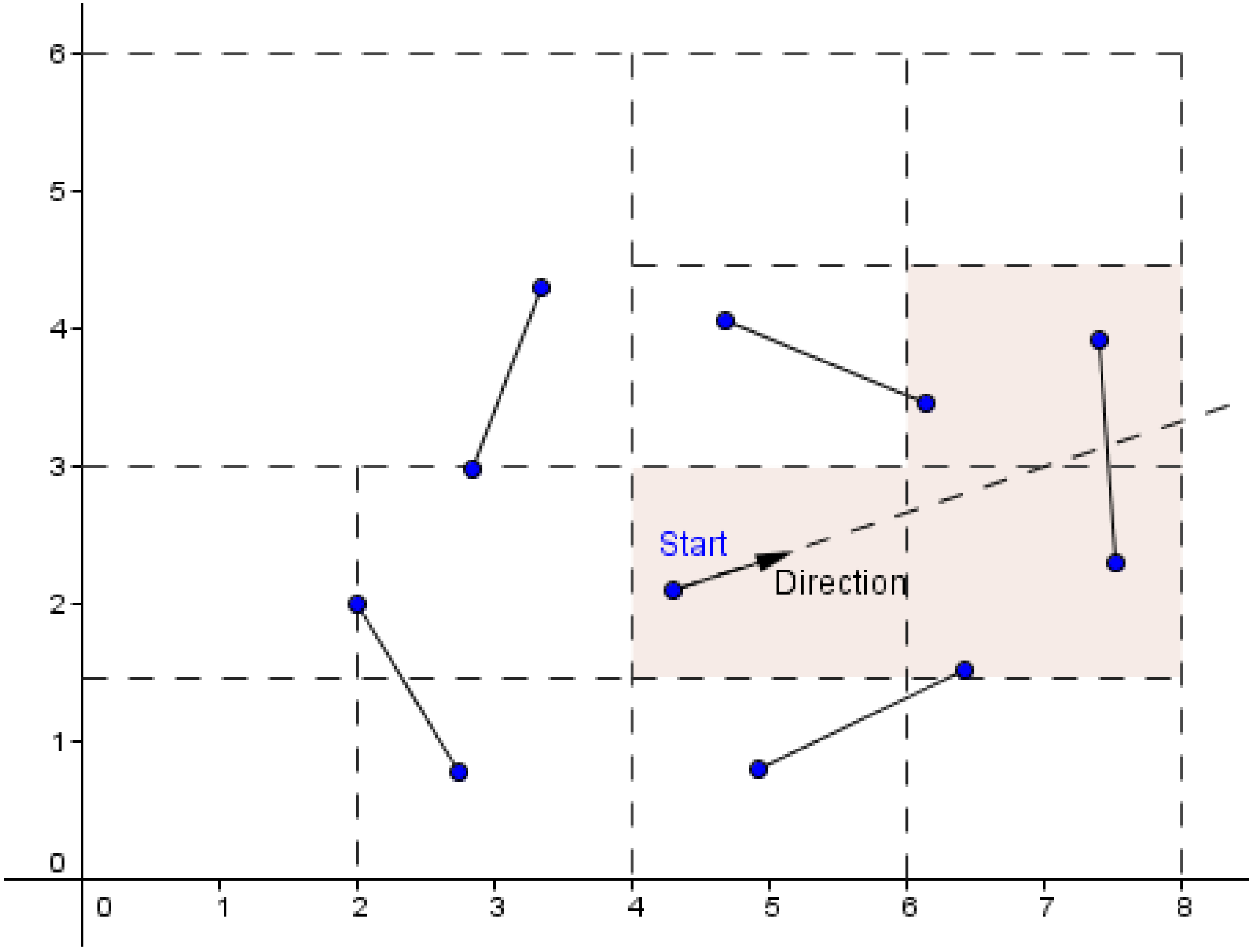
First Idea: Axis-Aligned Grid

- Most of the cells are empty...



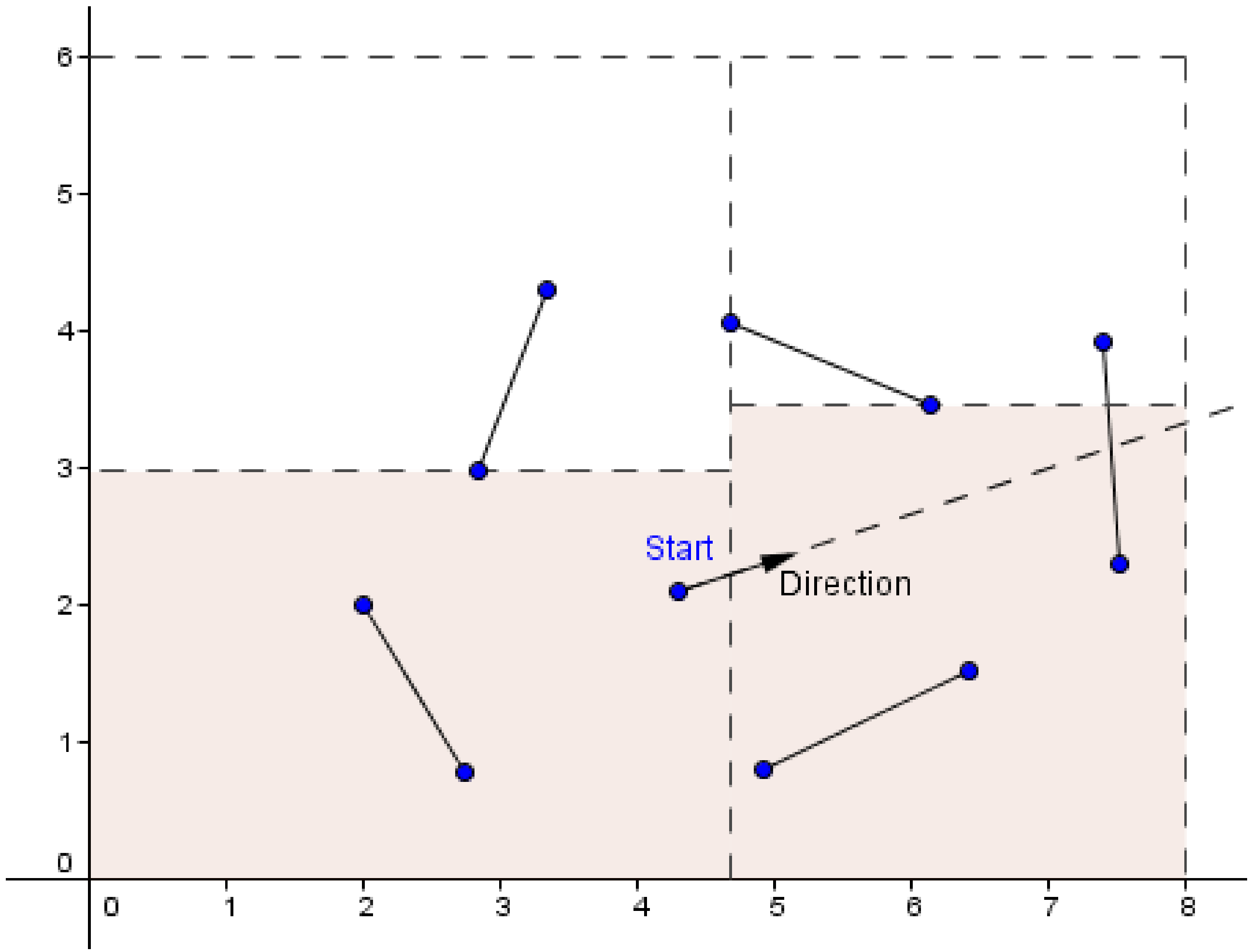
Second Idea: Quadtree / Octree

- Make the cells divide, if there are more objects inside them. Start with one cell for the entire scene.



Third Idea: K-D Tree

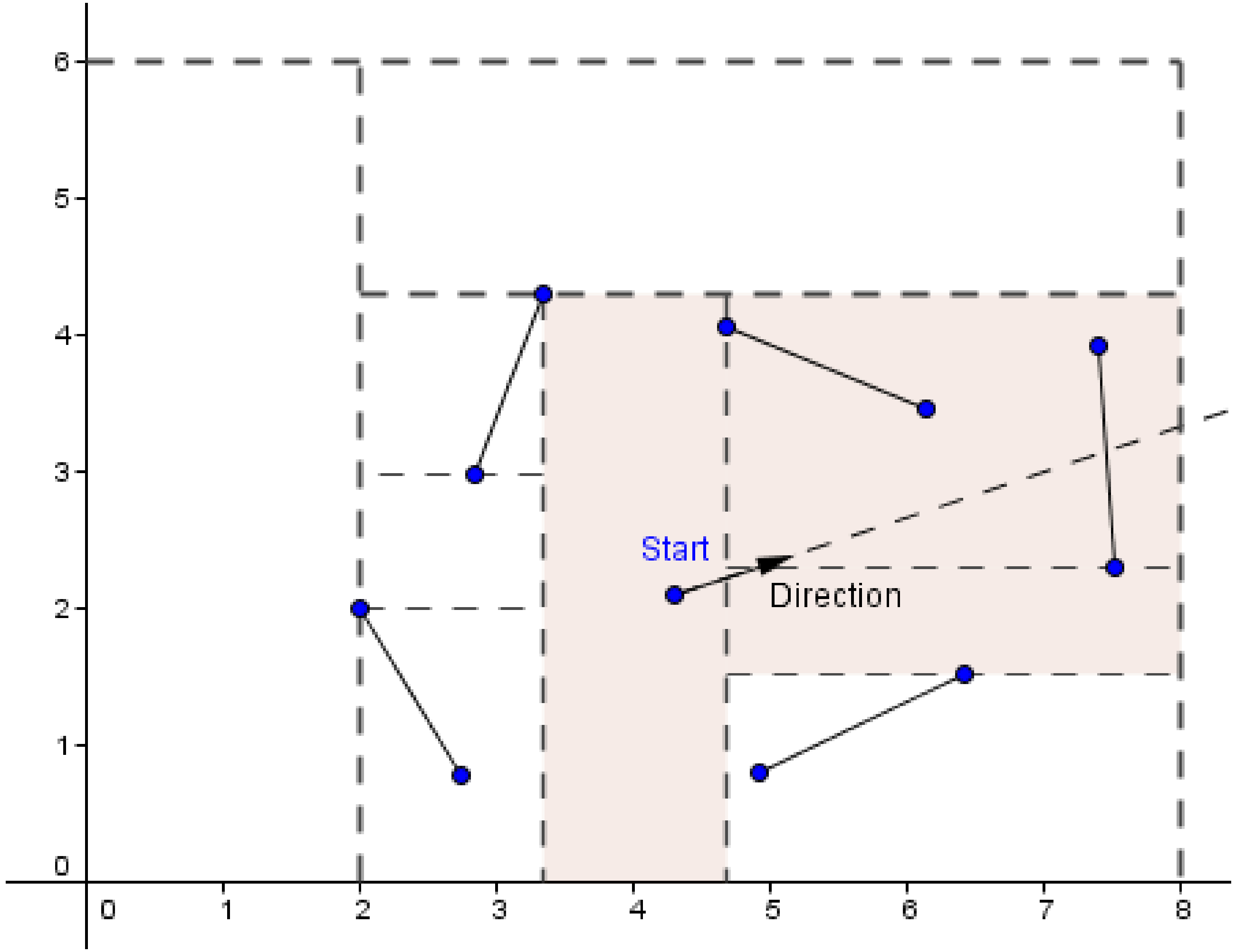
- Split according to the geometry. Traditionally by the median value.



No node will be empty.

Third Idea: K-D Tree

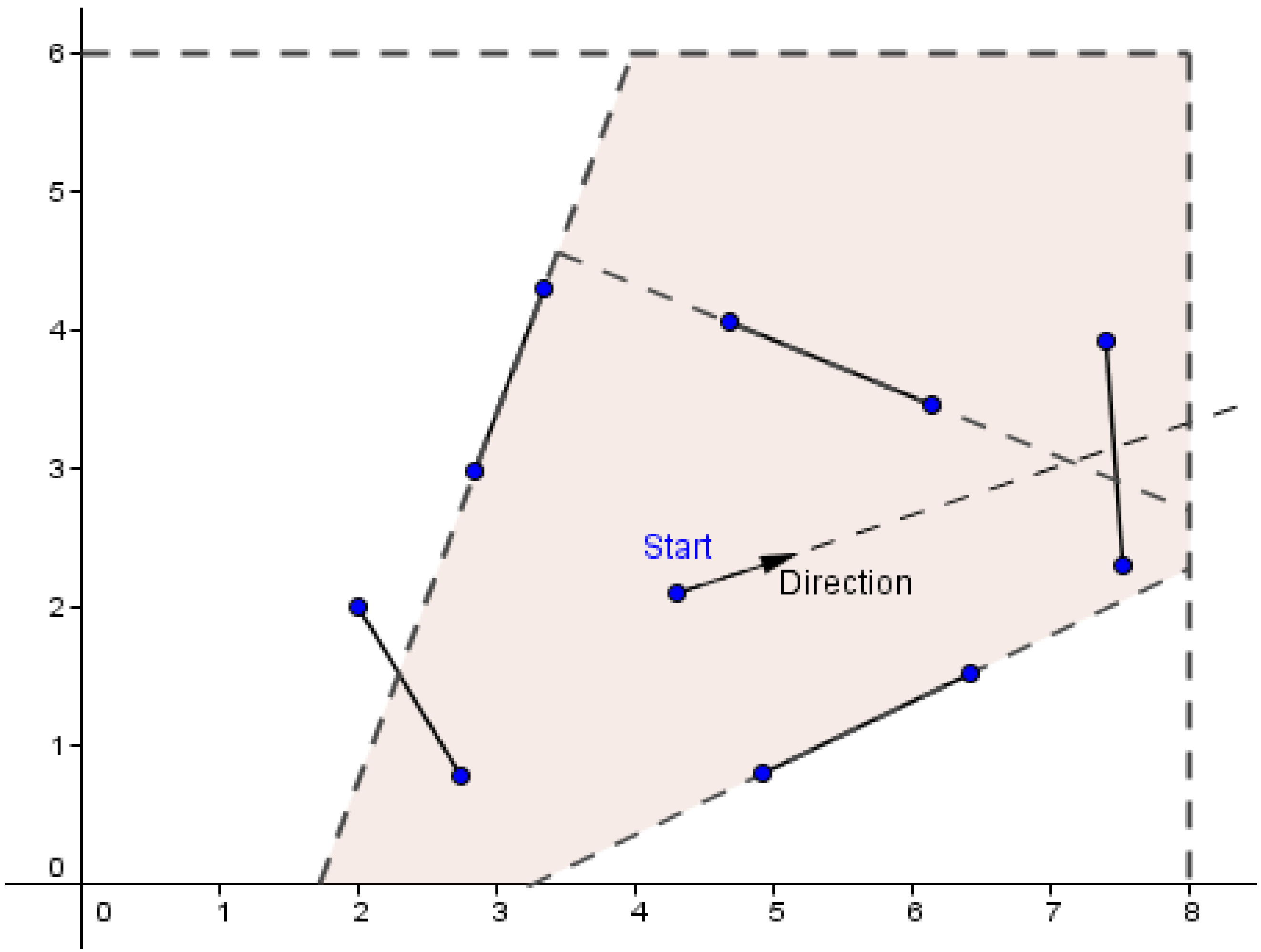
- Split with a rule to maximize the occurance of empty nodes.



Why is this good?

Fourth Idea: BSP Tree

- Binary Space Partitioning divides the space with existing polygons.

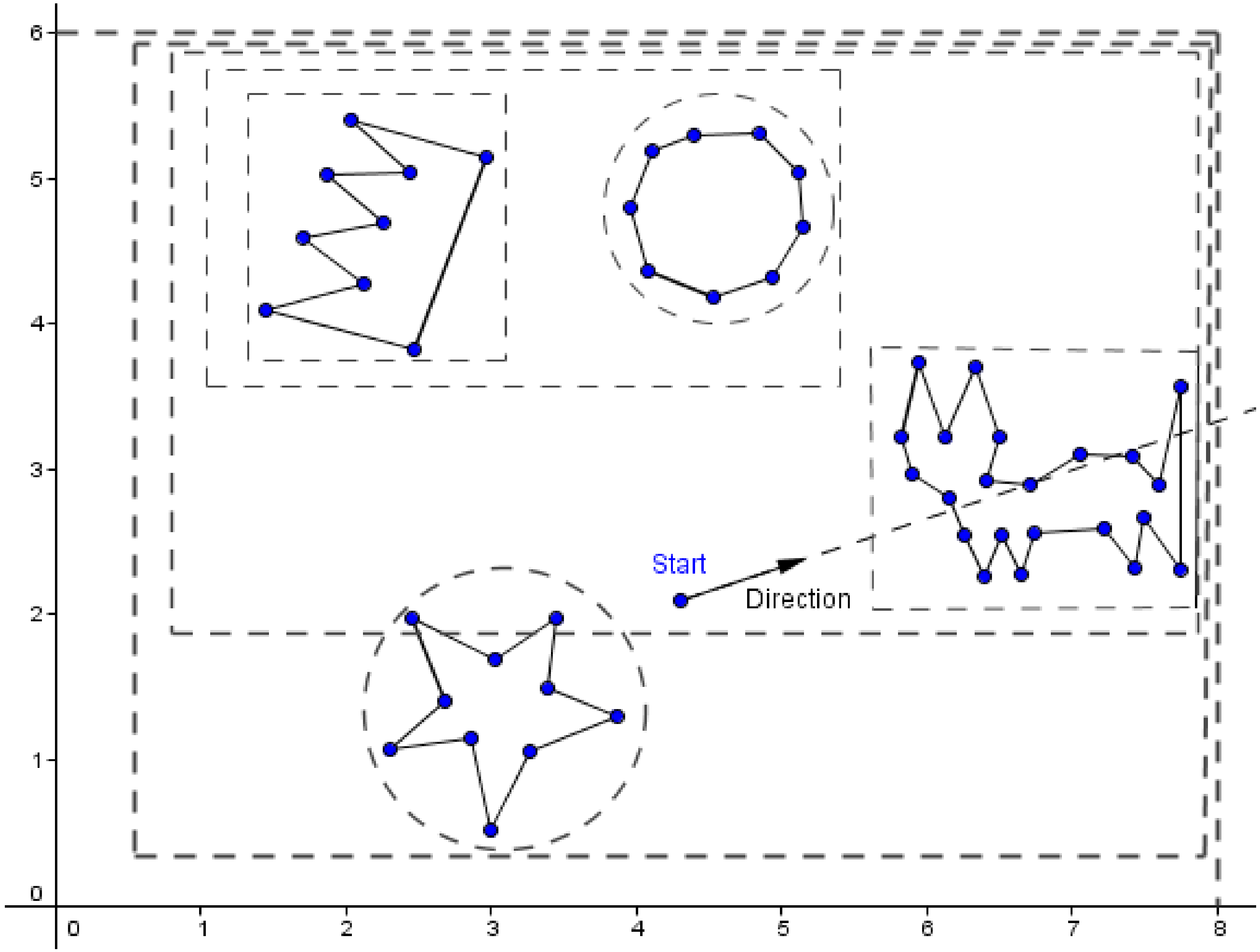


Not that useful for ray tracing.

Works well for geometry ordering (front to back).

Fifth Idea: BVH

- Bounding Volume Hierarchy – create a tree of bounding polygons around objects.



Bounding objects also useful for collision detection.

Axis-aligned bounding boxes.

Bounding spheres.

Space Partitioning

- Possible to combine different methods.
- Create structures, based on your own rules.
- Some better for dynamic, some for static scene.
- *Ray Tracing Acceleration Data Structures*: http://www.cse.iitb.ac.in/~paragc/teaching/2009/cs475/notes/accelerating_raytracing_sumair.pdf
- *Octree vs BVH*: http://thomasdiewald.com/blog/?p=1488

What did you found out today?

What more would you like to know?

Next time

Global Illumination