

# Computer Graphics

MTAT.03.015

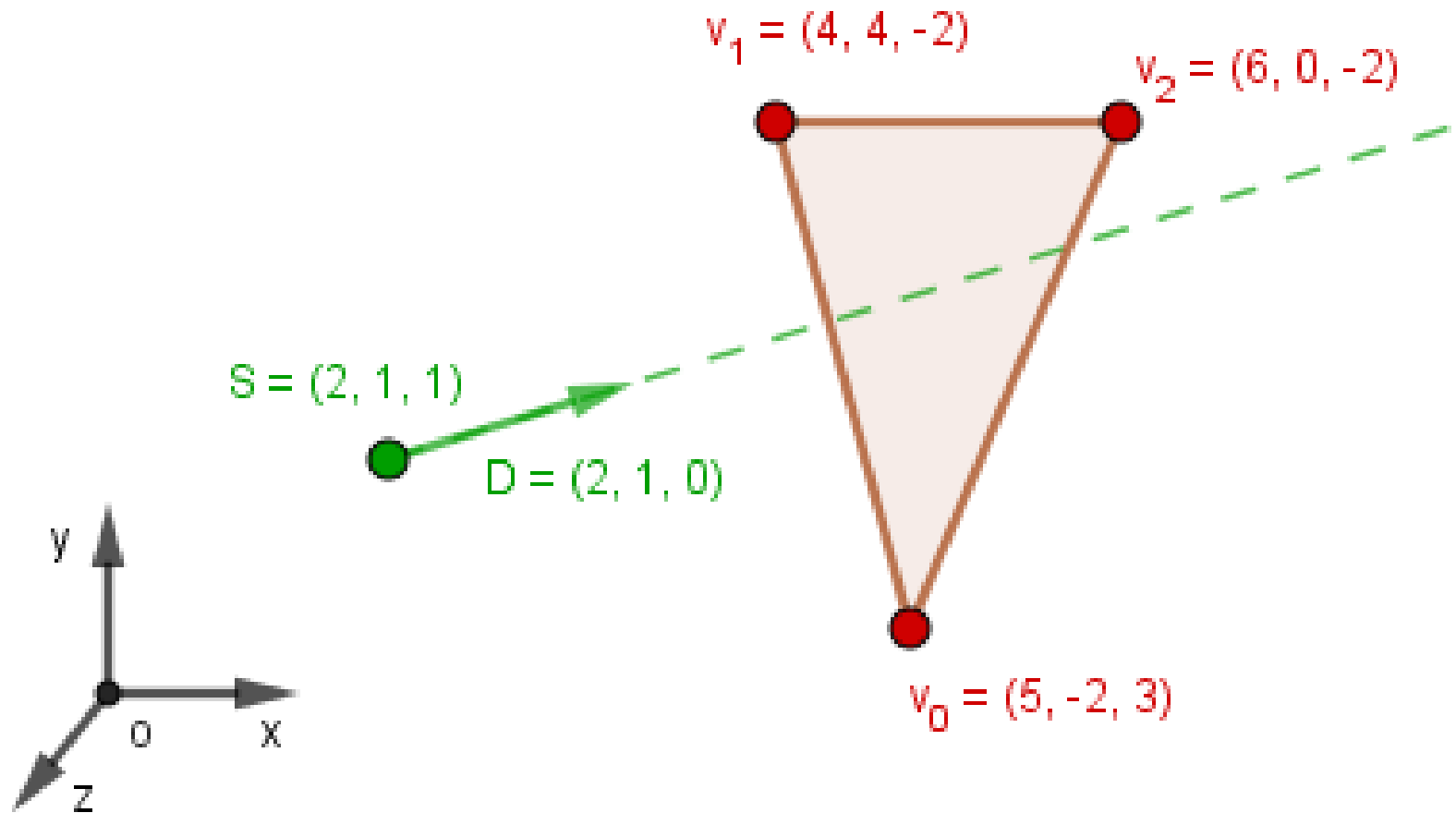
Raimond Tunnel



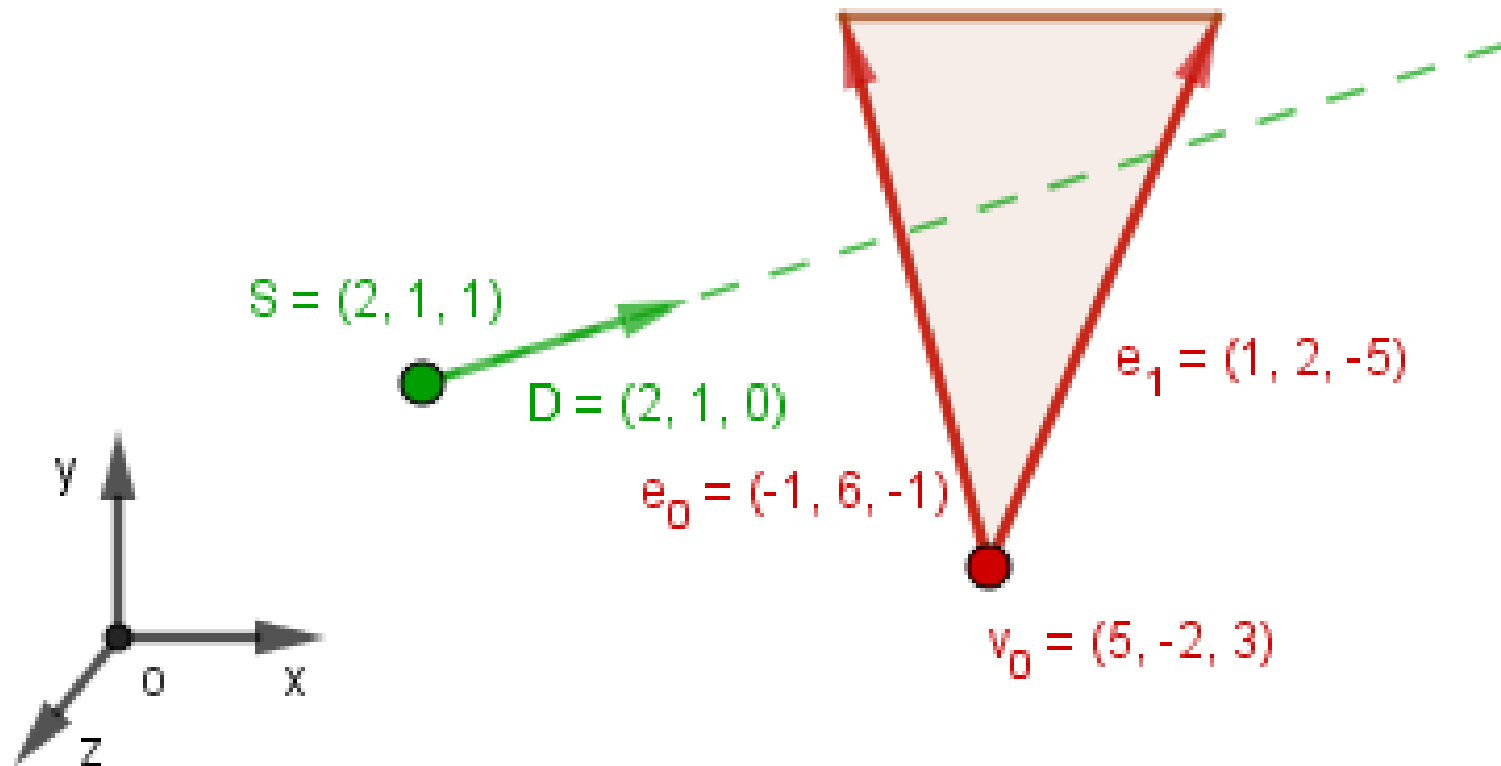
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# Ray Triangle Intersection Example



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*Ray*( $t$ )

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

# Ray Triangle Intersection Example

*Ray*( $t$ )

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

*Triangle*( $u, v$ )

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} \quad e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$\mathit{Ray}(t) = \mathit{Start} + t \cdot \mathit{Direction} = v_0 + u \cdot e_0 + v \cdot e_1 = \mathit{Triangle}(u, v)$$

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$$\mathit{Ray}(t) = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} + u \cdot \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + v \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \mathit{Triangle}(u, v)$$

# Ray Triangle Intersection Example

$$\mathit{Ray}(t) = \mathit{Start} + t \cdot \mathit{Direction} = v_0 + u \cdot e_0 + v \cdot e_1 = \mathit{Triangle}(u, v)$$

$$\mathit{Ray}(t) = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} + u \cdot \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + v \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \mathit{Triangle}(u, v)$$

$$u \cdot \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + v \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} - t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$



# Ray Triangle Intersection Example

$$u \cdot \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + v \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} - t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & -2 \\ 6 & 2 & -1 \\ -1 & -5 & 0 \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$u \cdot \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + v \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} - t \cdot \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & -2 \\ 6 & 2 & -1 \\ -1 & -5 & 0 \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ t \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

This is the system of linear equations we are solving.

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

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$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\hat{P} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix} = 62$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

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$$P = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\hat{P} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix} = 62$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$62 > \sim 0 \rightarrow$  Ray is not parallel nor approaching from the back!  
We can continue!

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$u = \frac{\begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix}}{62} = \frac{39}{62} \approx 0.63$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$



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$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$u = \frac{\begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 10 \\ 3 \end{pmatrix}}{62} = \frac{39}{62} \approx 0.63$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$\sim 0 < 0.63 < 1 \rightarrow$  We can continue!

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

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$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} -3 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}$$

$$v = \frac{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}}{62} = \frac{17}{62} \approx 0.27$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

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$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

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$$v = \frac{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}}{62} = \frac{17}{62} \approx 0.27$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$\sim 0 < 0.27 < 1 \rightarrow$  We can continue!

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{17}{62} + \frac{39}{62} = \frac{56}{62} \approx 0.90$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\frac{17}{62} + \frac{39}{62} = \frac{56}{62} \approx 0.90$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$\sim 0 < 0.90 < 1 \rightarrow$  We can continue!

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$t = \frac{\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}}{62} = \frac{82}{62} \approx 1.32$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$t = \frac{\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ -1 \\ -15 \end{pmatrix}}{62} = \frac{82}{62} \approx 1.32$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

$\sim 0 < 1.32 \rightarrow$  We can continue!



# Ray Triangle Intersection Example

$$S = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ t \end{pmatrix} = \begin{pmatrix} \frac{39}{62} \\ \frac{17}{62} \\ \frac{82}{62} \end{pmatrix} = \begin{pmatrix} \sim 0.63 \\ \sim 0.27 \\ \sim 1.32 \end{pmatrix}$$

$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

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$$v_0 = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$$

$$e_0 = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

System is solved!

Return  $t$  or find the hit point and return that.