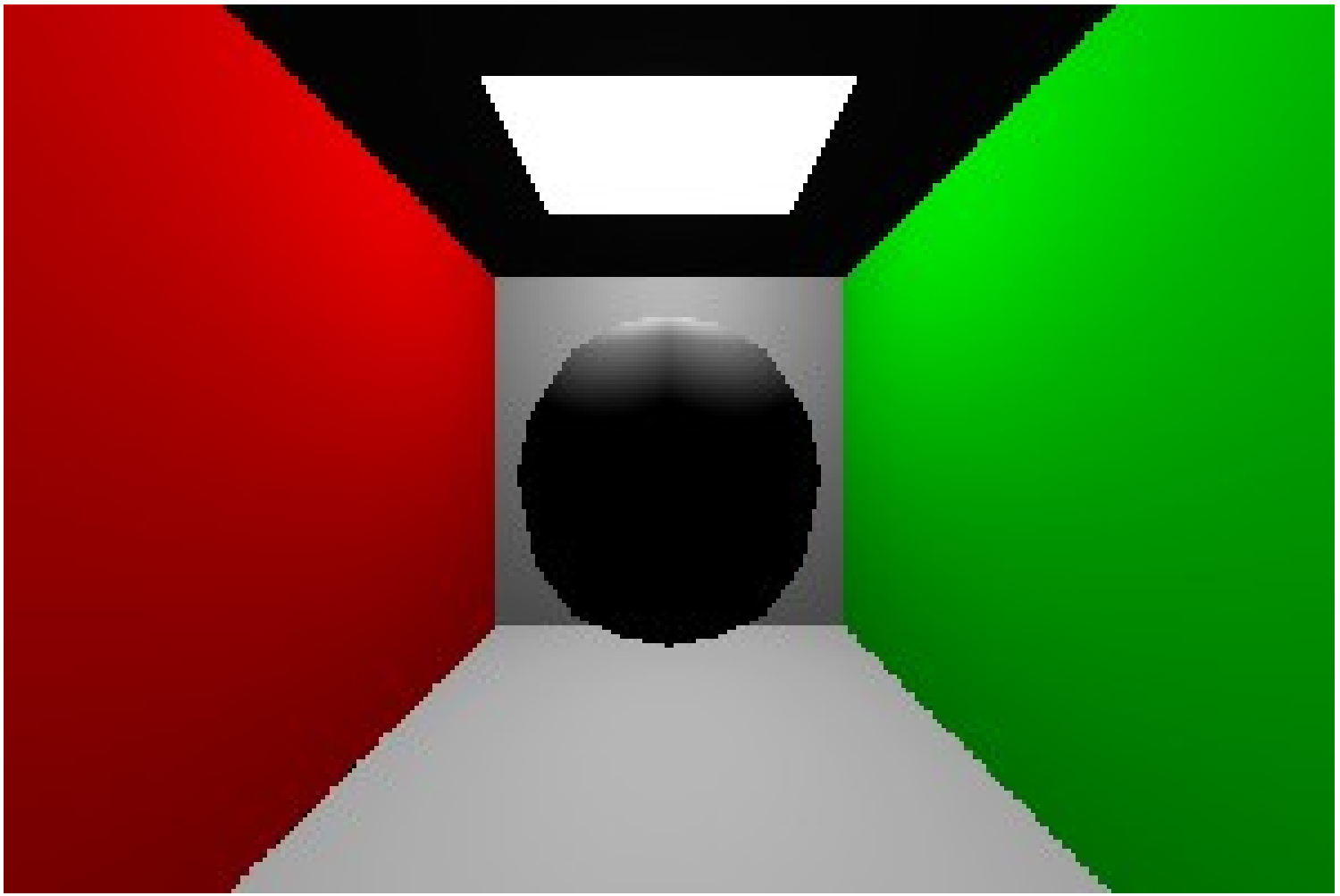
Computer Graphics

MTAT.03.015

Raimond Tunnel

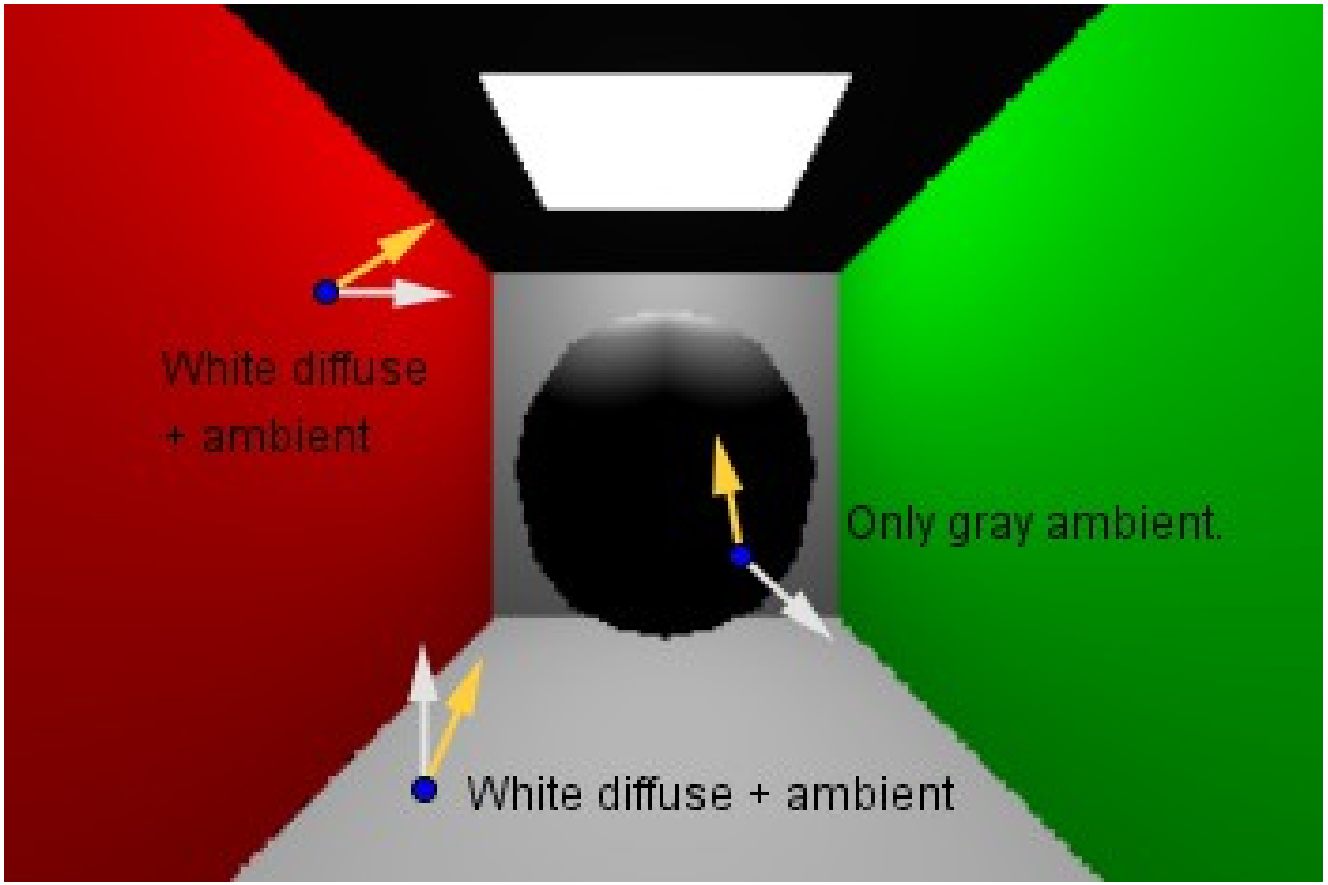
Study IT in .ee

The Road So Far...



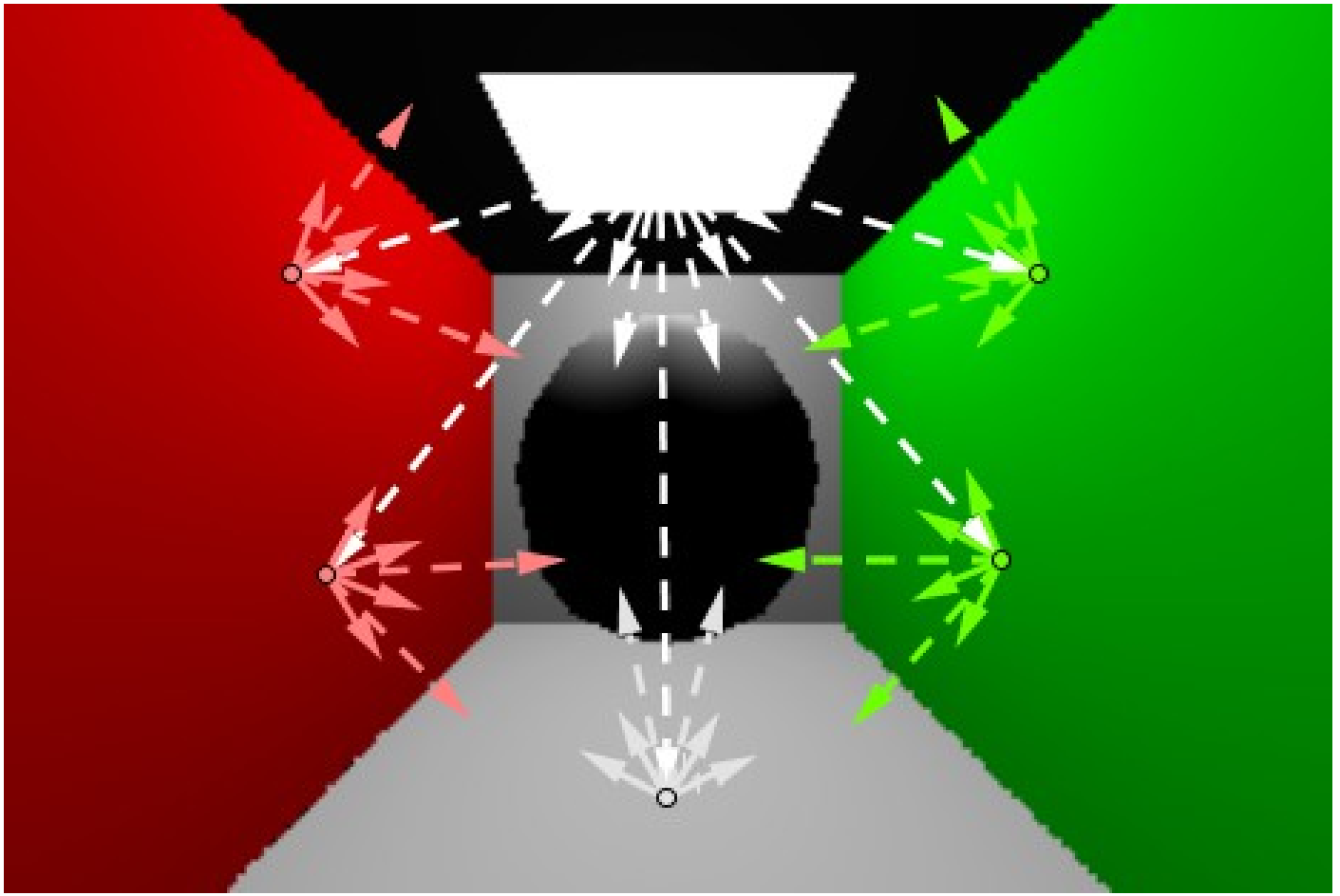
Direct / Local Illumination

- Apply a lighting model for a fragment. We consider only the light source.



Indirect Lighting

- Not only does the light source illuminate the scene. The scene itself illuminates the scene.



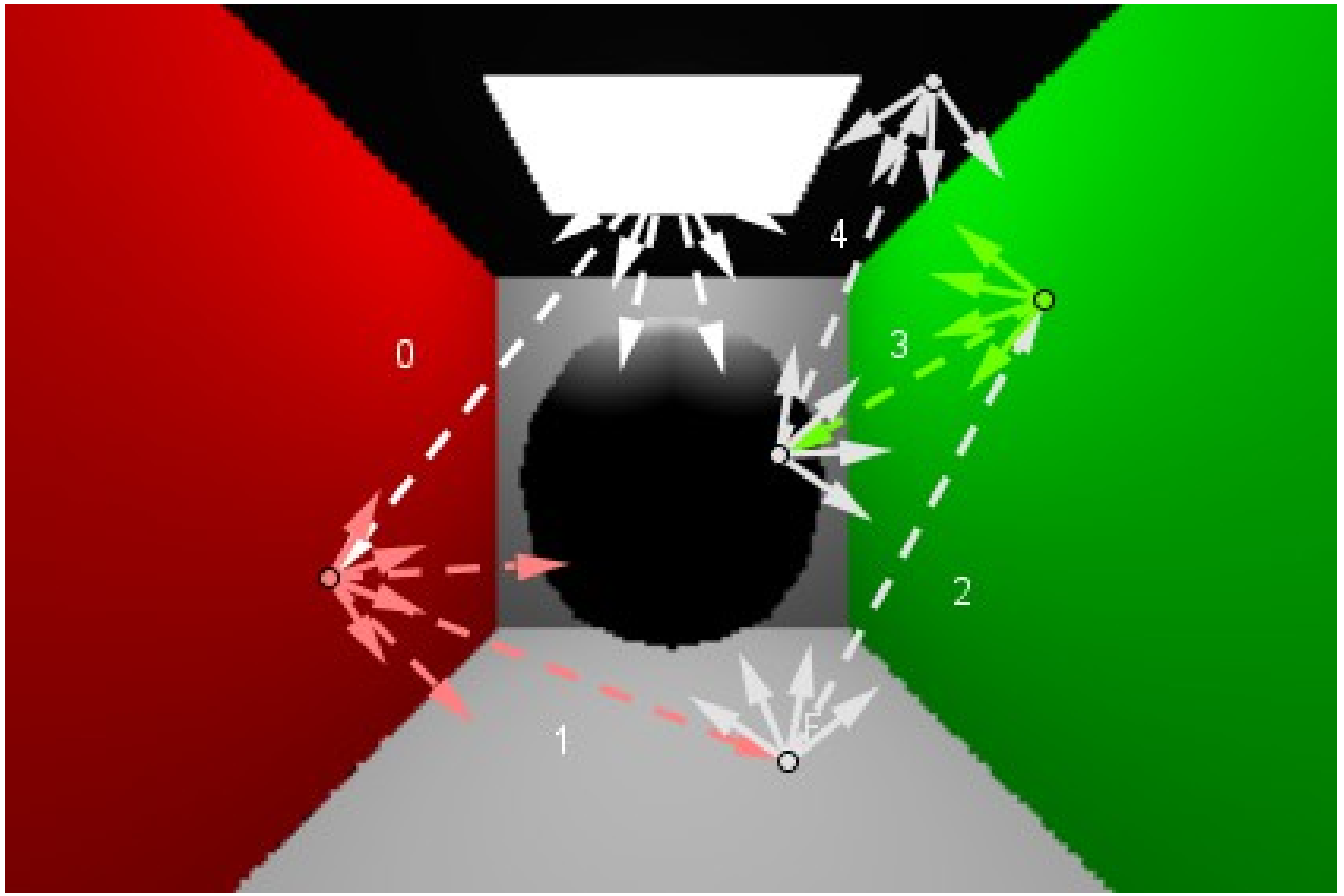
Cornell Box

- This scene is called the **Cornell Box scene**.
- It is often used to test global illumination.
- Left wall red, right wall green.
- Other walls are white / gray.
- Light source in the ceiling.
- Objects inside the room.

By LockRickard:
http://lockrikard.deviantart.com/art/Minecraft-Cornell-Box-356384680

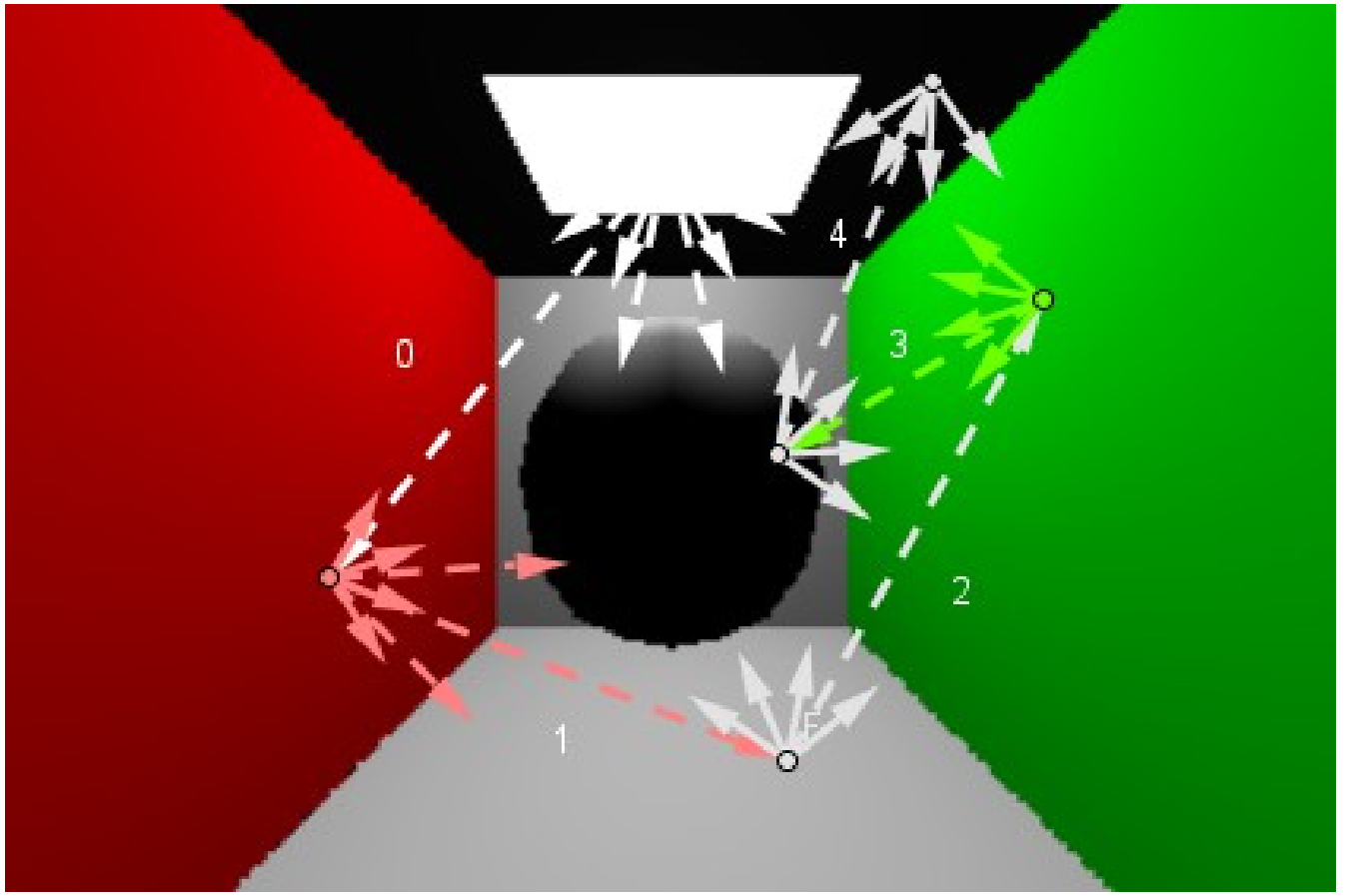
Indirect Lighting

- The light bounces around more than once...

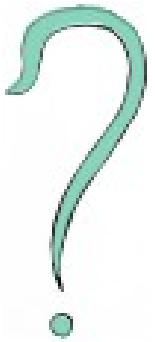


Indirect Lighting

- The light bounces around more than once...

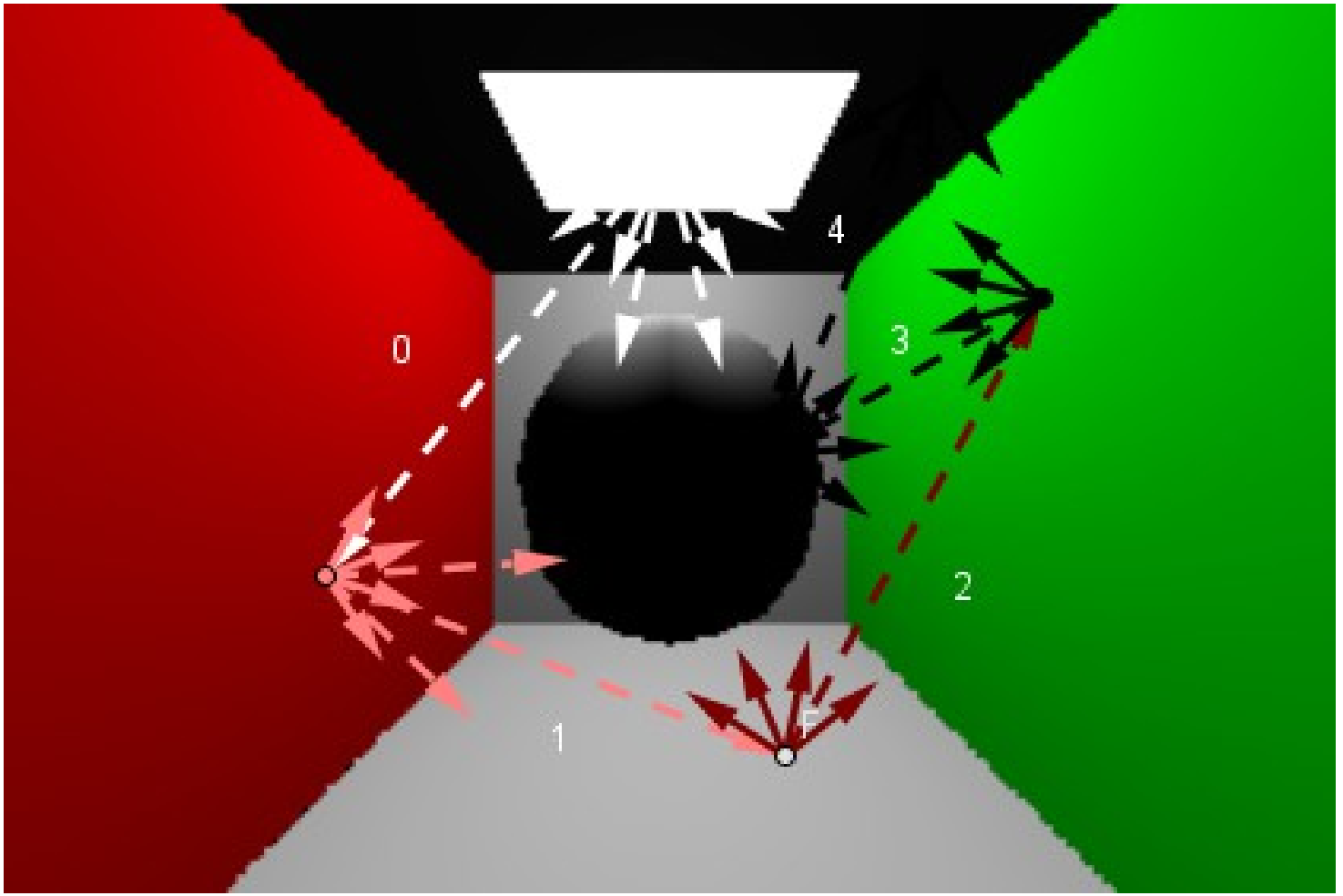


- What is wrong with this picture?



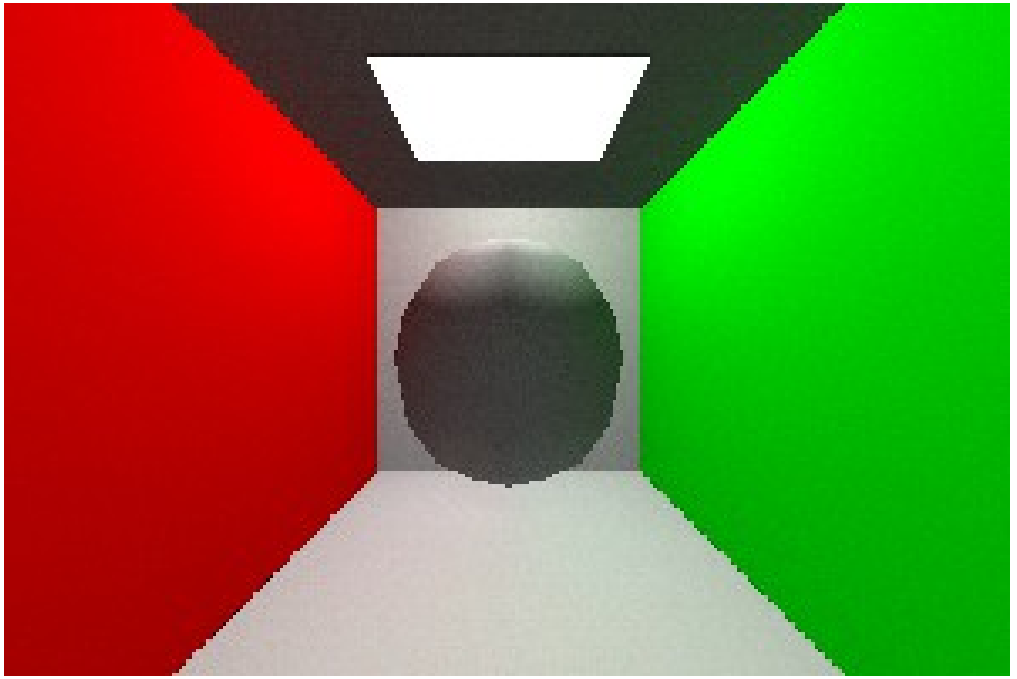
Indirect Lighting

- It is more like...



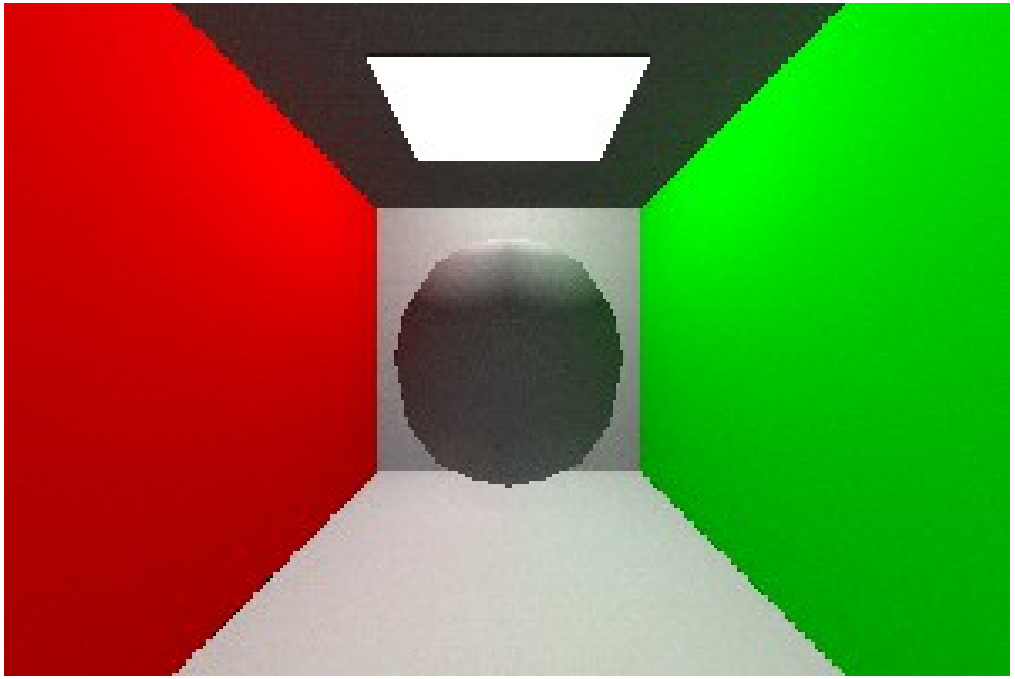
Global Illumination

- Take into account both direct (from the light source) and **indirect lighting**.



Global Illumination

- Take into account both direct (from the light source) and **indirect lighting**.
- Alternatively, think of all the surfaces as light sources.

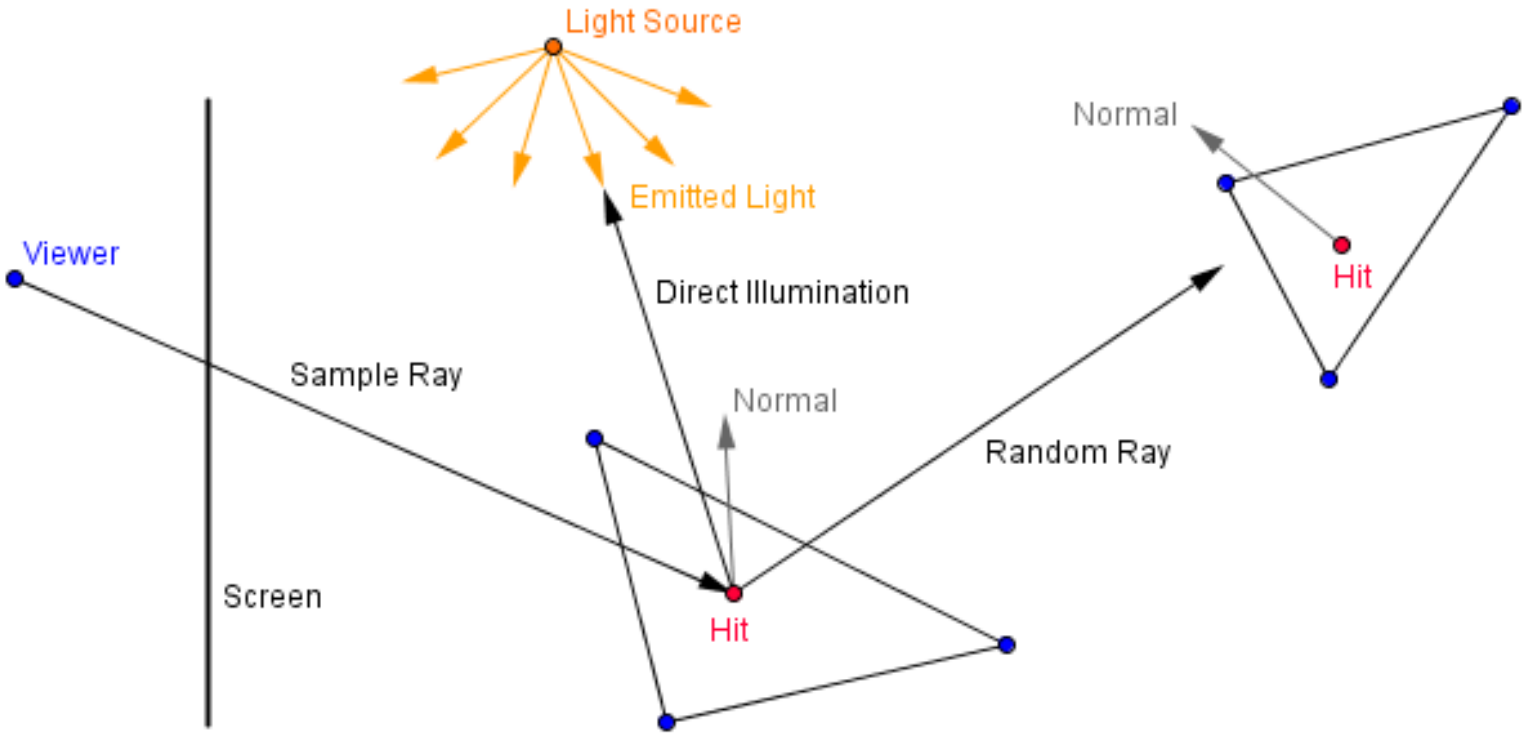


Path Tracing (with Direct Illum.)

- Algorithm based on ray trace rendering.

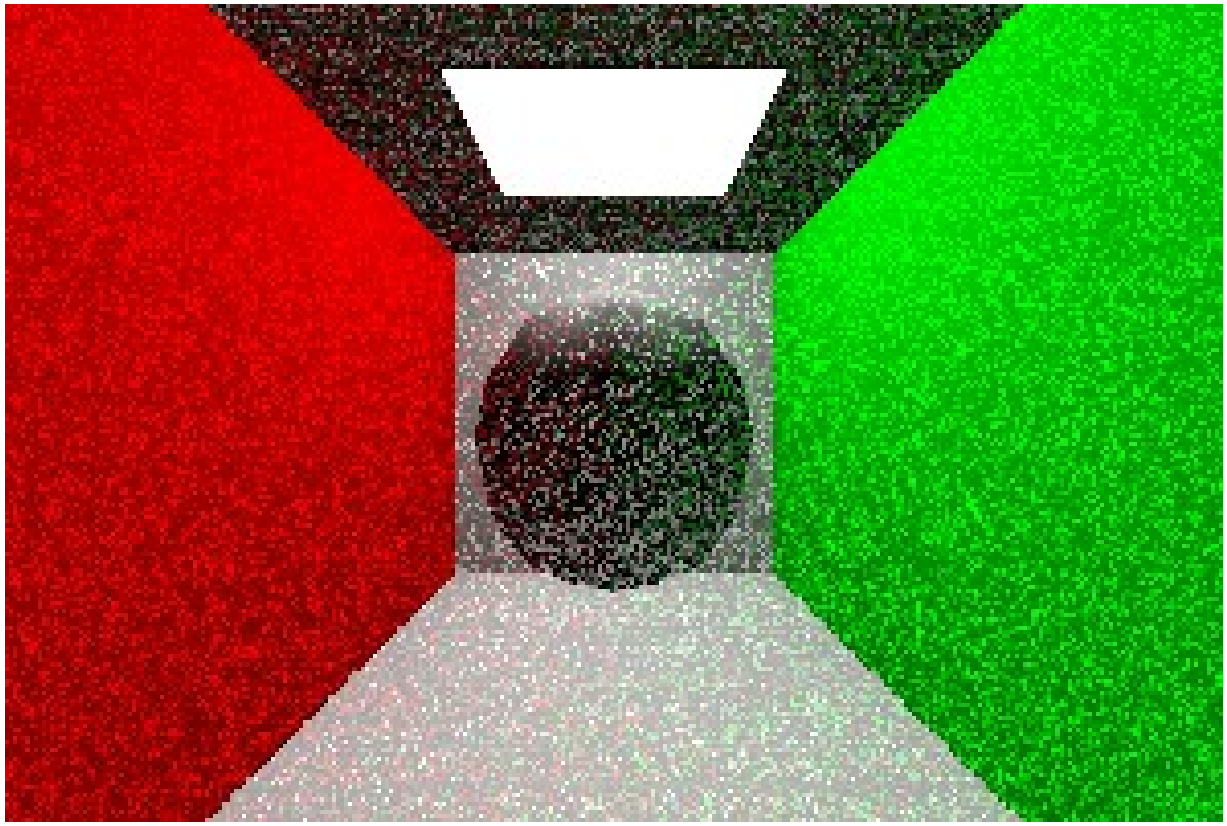
Path Tracing (with Direct Illum.)

- Algorithm based on ray trace rendering.
- For each hit, send random ray(s) to sample indirect illumination from the scene.



Path Tracing (with Direct Illum.)

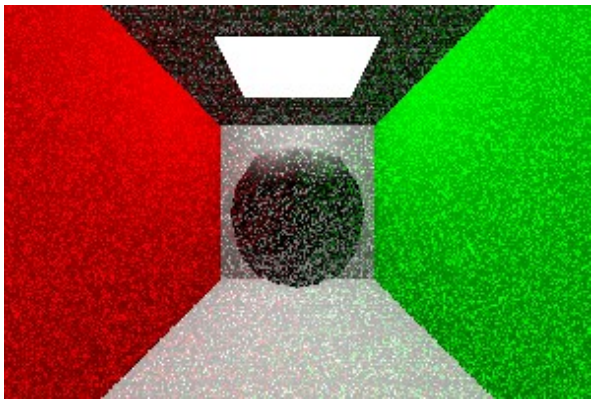
- Did just that, got this result:



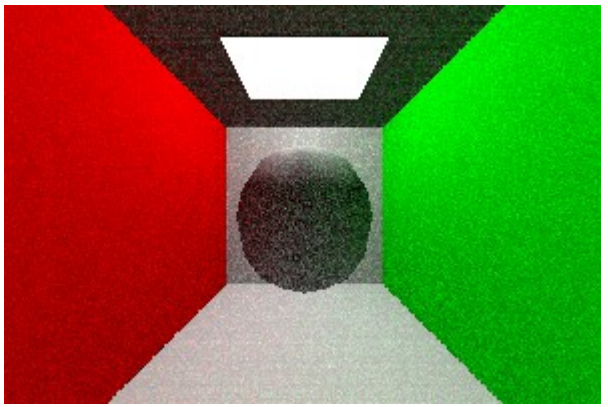
- That is correct, it just one sample.

Path Tracing (with Direct Illum.)

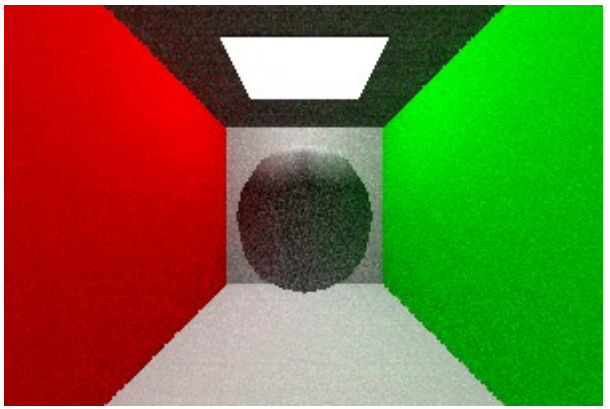
- Because the rays are random, we need to sample a lot of them, and average all results.



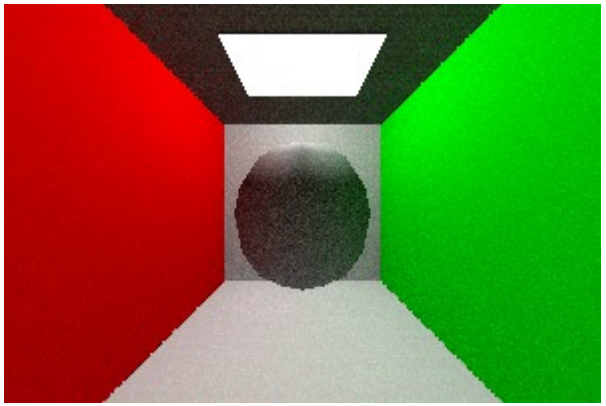
1 sample



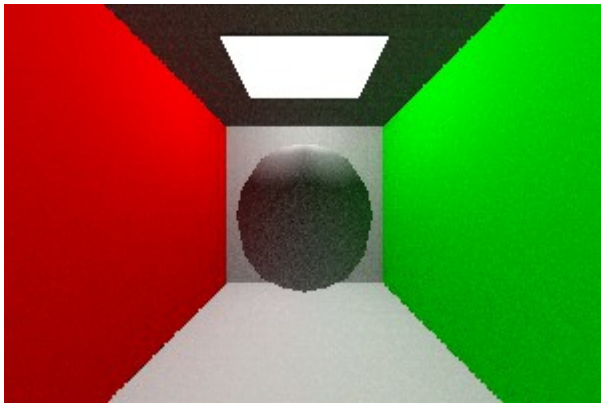
5 samples



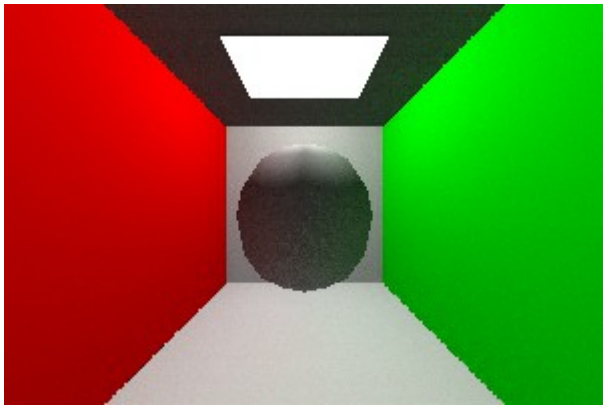
10 samples



20 samples



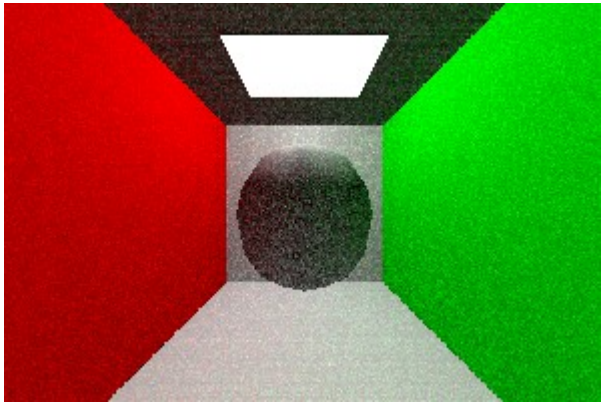
40 samples



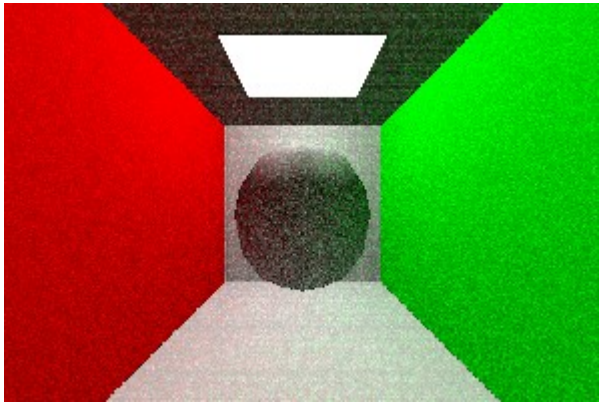
80 samples

Path Tracing (with Direct Illum.)

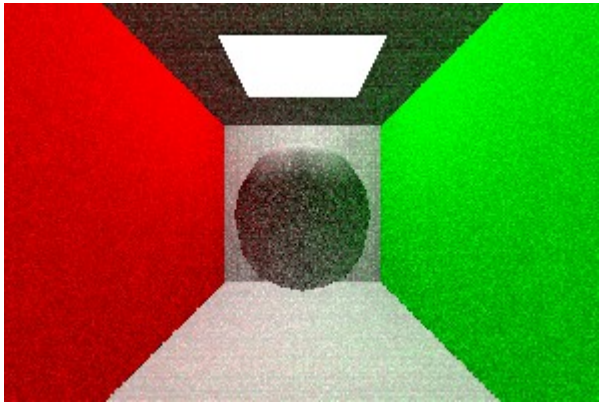
- That was just one bounce. We can do more.



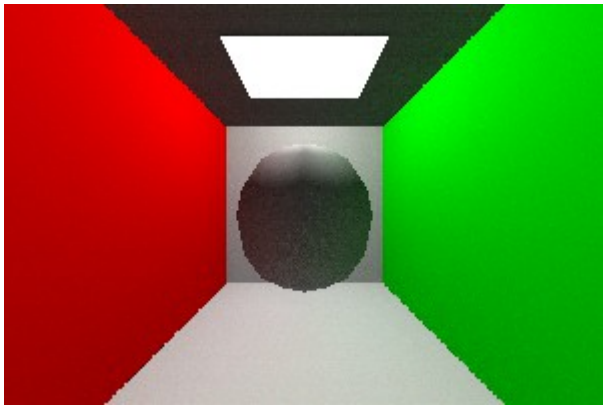
5 samples, 1 bounce



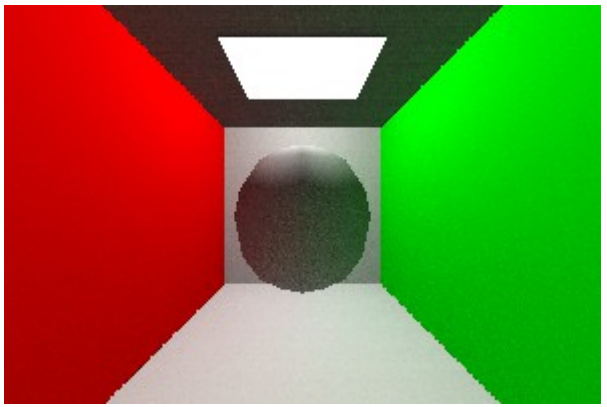
5 samples, 2 bounces



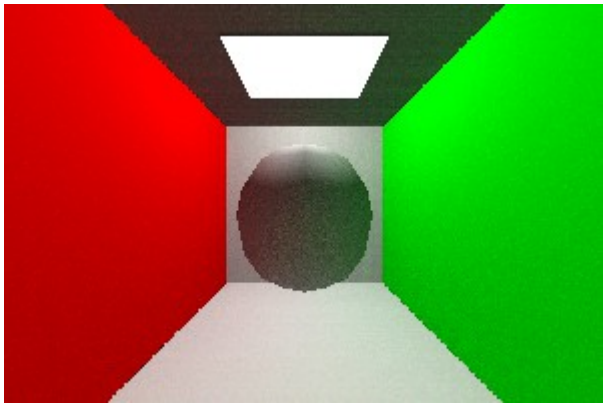
5 samples, 3 bounces



80 samples, 1 bounce



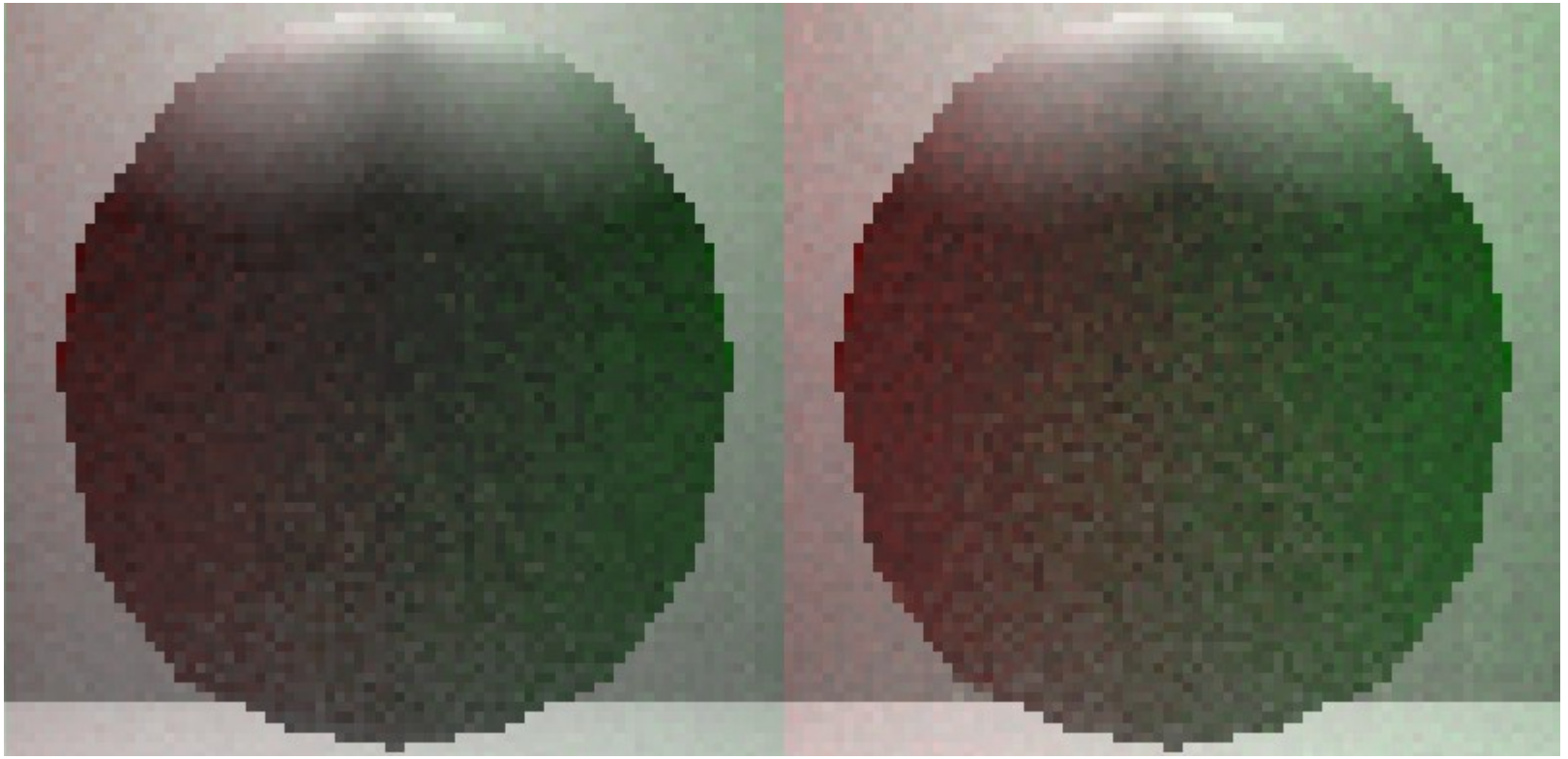
80 samples, 2 bounces



80 samples, 3 bounces

Path Tracing (with Direct Illum.)

- There are differences.



80 samples, 1 bounce versus 3 bounces

Path Tracing

- There are other path tracing techniques also.

Path Tracing

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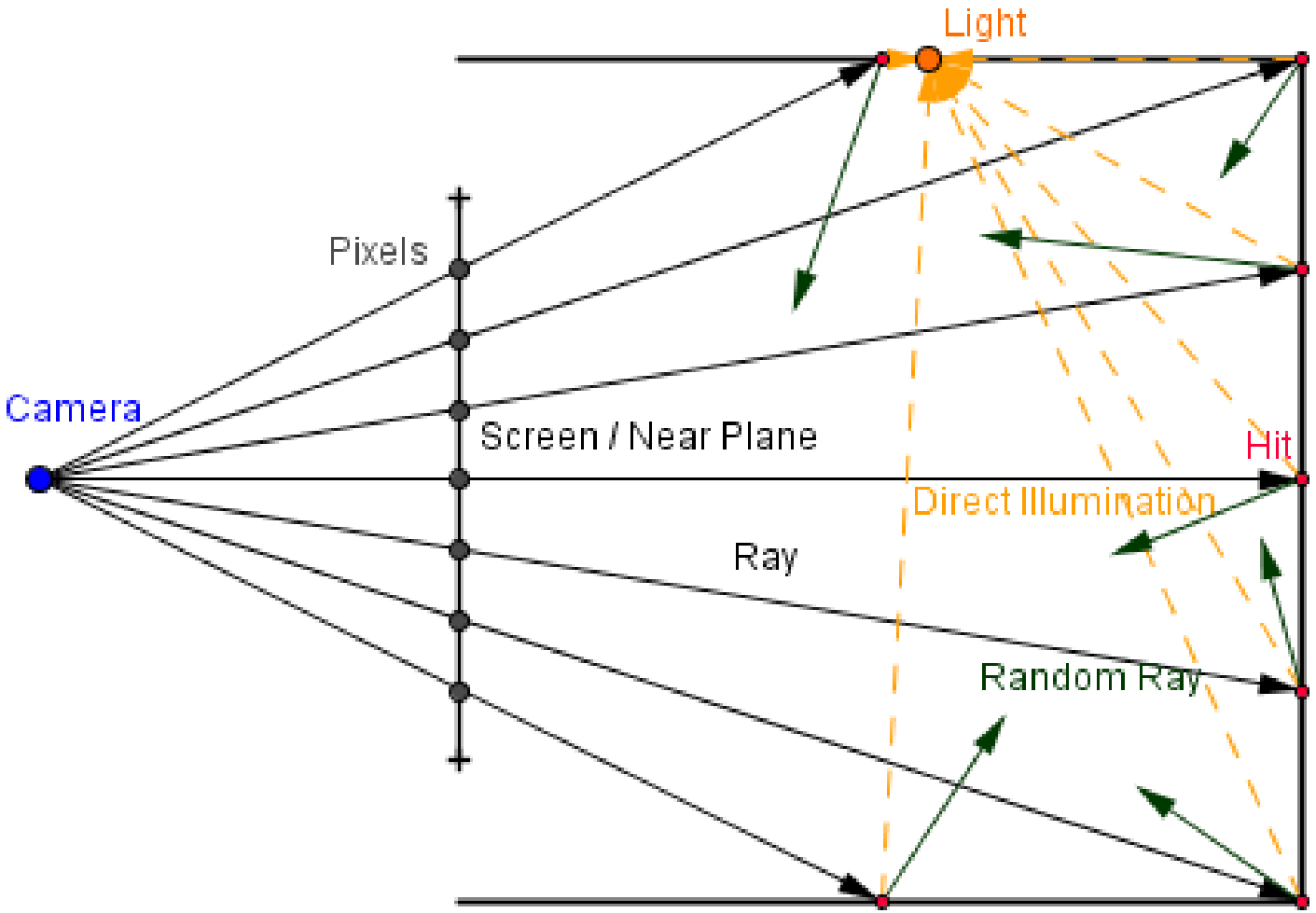
- Without direct illumination:
  - Only bounce, until we reach the light source.
  - Light source intensity should be more than 1.
  - Direct into the light source at a random bounce.

Path Tracing

- There are other path tracing techniques also.
- Without direct illumination:
  - Only bounce, until we reach the light source.
  - Light source intensity should be more than 1.
  - Direct into the light source at a random bounce.
- Create a number of random rays at the first bounce.

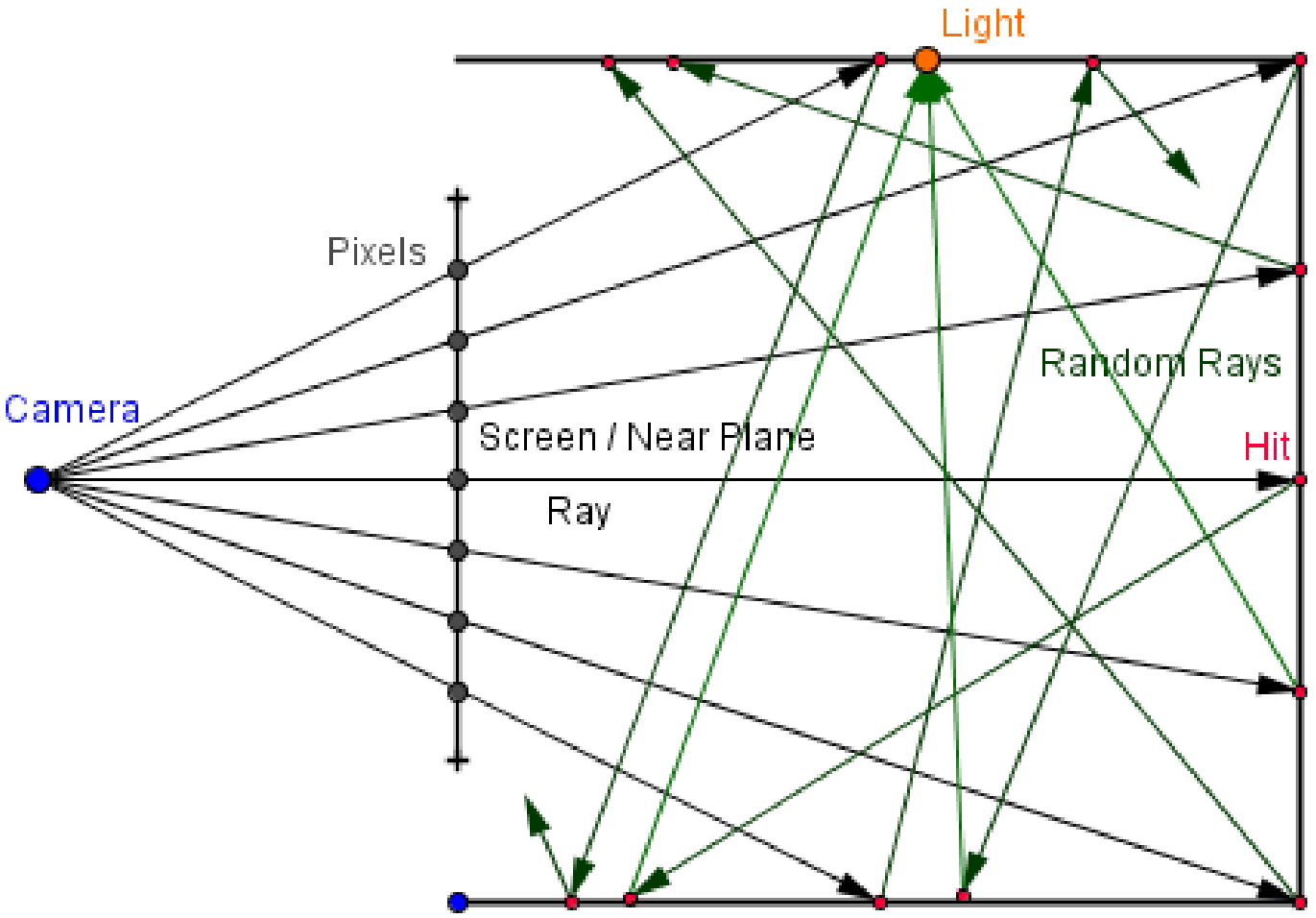
Path Tracing

- With direct illumination



Path Tracing

- Without direct illumination (direct rays to light, after a random number of bounces).

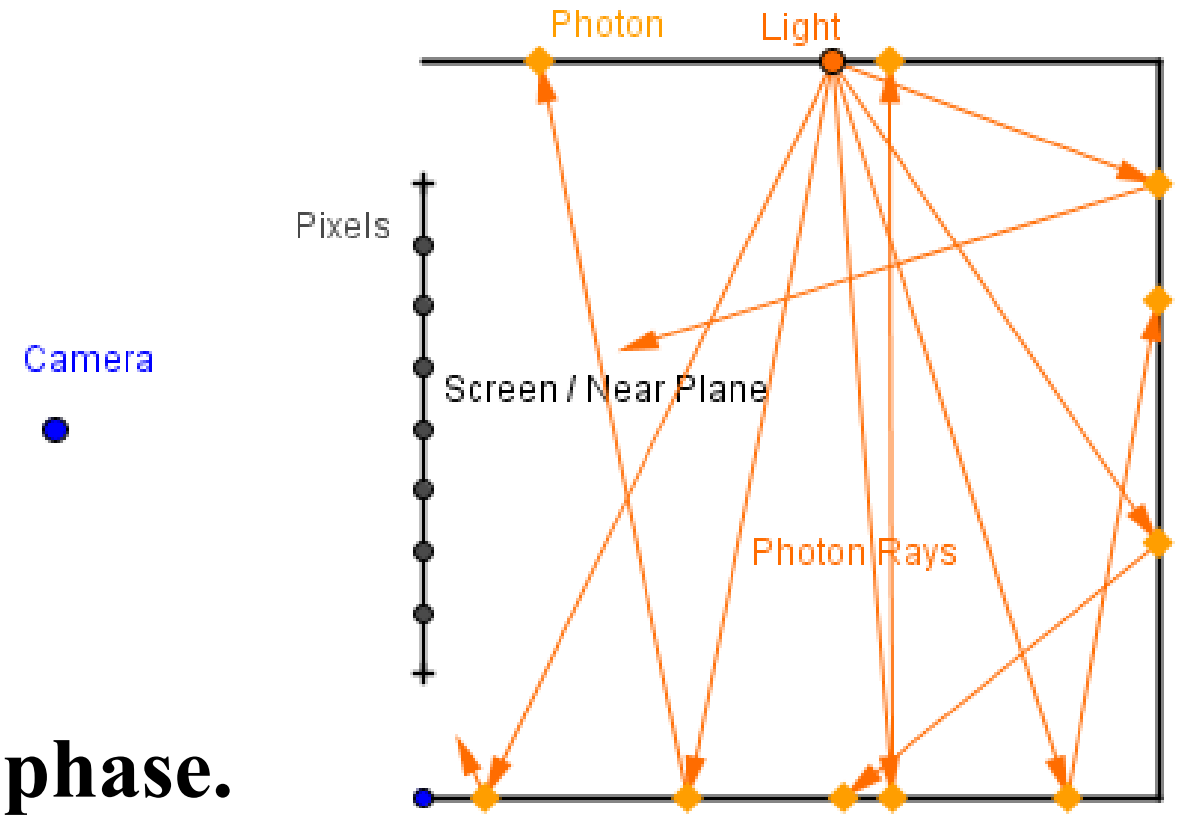


Path Tracing

- Examples:
  - Thorough project and overview: https://github.com/erichlof/THREE.js-PathTracing-Renderer
  - Nice interactive version: http://madebyevan.com/webgl-path-tracing/
  - Non-parallel implementation on CPU: https://github.com/hunterloftis/pathtracer
  - Shadertoy search (slow on laptops): https://www.shadertoy.com/results?query=tag%3Dpathtracer

Photon Mapping

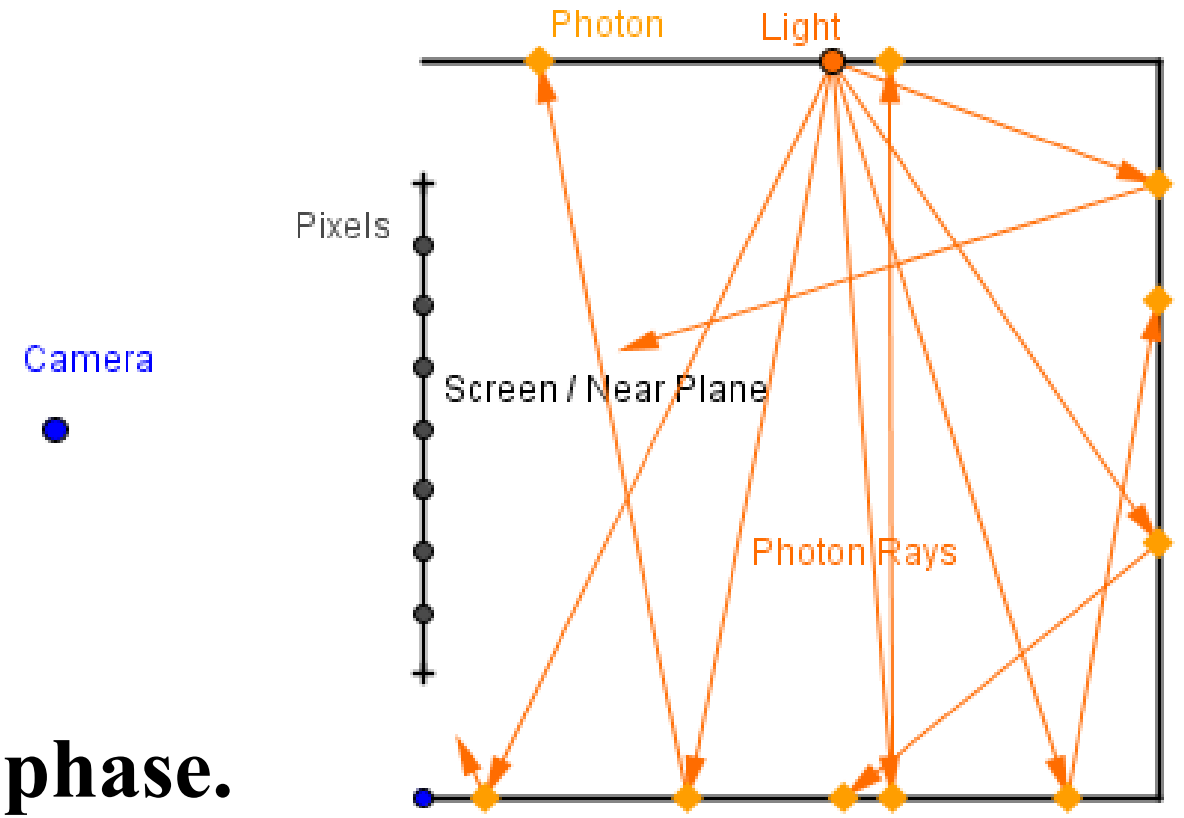
- First shoot rays from the light source.



Construction phase.

Photon Mapping

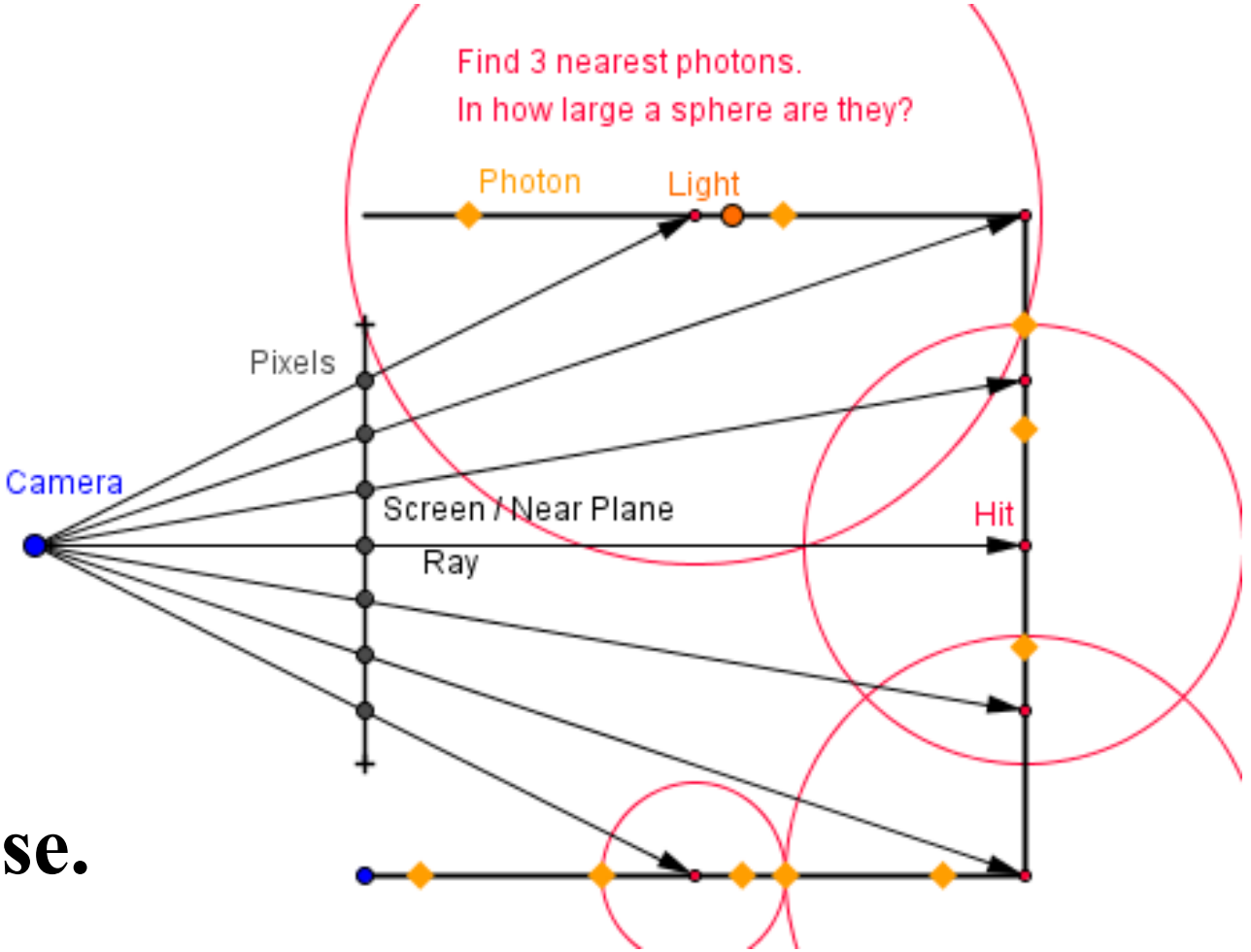
- First shoot rays from the light source.
- Create a map of photons, that those rays distribute.



Construction phase.

Photon Mapping

- Then shoot rays from the camera, find nearest photons for the hit points.



Gathering phase.

Photon Mapping

- Illumination of the surface can be estimated, by considering the nearest photons, divided by the area of the minimum sphere covering those.

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Photon Mapping

- Illumination of the surface can be estimated, by considering the nearest photons, divided by the area of the minimum sphere covering those.
- Some illumination (direct, specular) can be calculated without the photon map.
- Gathering phase is much slower, then the construction.
- How to store the photon map for fast nearest neighbour search?

More Cool Stuff

- Photon Mapping (In Estonian) by Hendrik Eerikson:
  http://tume-maailm.pri.ee/ylikool/CG/articles/Kiirtej%C3%A4litusalgoritmi%20ja%20footonkaardistamise%20praktiline%20rakendus%20Pythonis.pdf
  + http://tume-maailm.pri.ee/ylikool/CG/2017/slides/12/PhotonMapping.png
- Photon mapping:
  https://github.com/erlandranvinge/photons
- Real-time GPU Path Tracing in Quake 2
  https://www.youtube.com/watch?v=x19sIltR0qU
- WebGL Path Tracing
  http://madebyevan.com/webgl-path-tracing/

What important things you learned?

What more would you like to know?

Next time: Global Illumination continued

The Rendering Equation

- *The rendering equation* –
  Jim Kajiya, SIGGRAPH 1986.
- https://doi.org/10.1145/15922.15902
- ~900 citations via CiteSeerX

Jim Kajiya

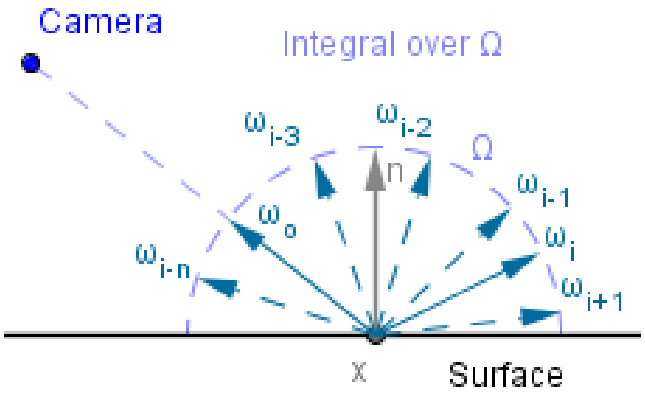
Jim Kajiya in his office at Microsoft Research in 2009.

Alma mater	University of Utah
Known for	Development of the rendering equation
	Scientific career
Fields	Computer graphics
Institutions	Caltech

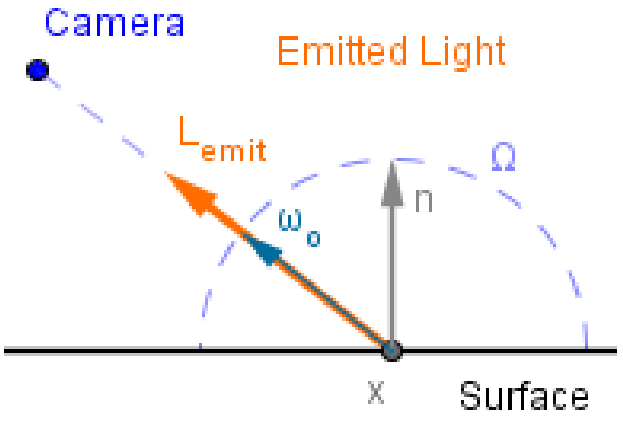
- https://en.wikipedia.org/wiki/Rendering_equation
- https://en.wikipedia.org/wiki/Jim_Kajiya
- http://www.cse.chalmers.se/edu/year/2011/course/TDA361/2007/rend_eq.pdf
- Who else do you recognize here:
  https://en.wikipedia.org/wiki/Computer_graphics#Other_pioneers

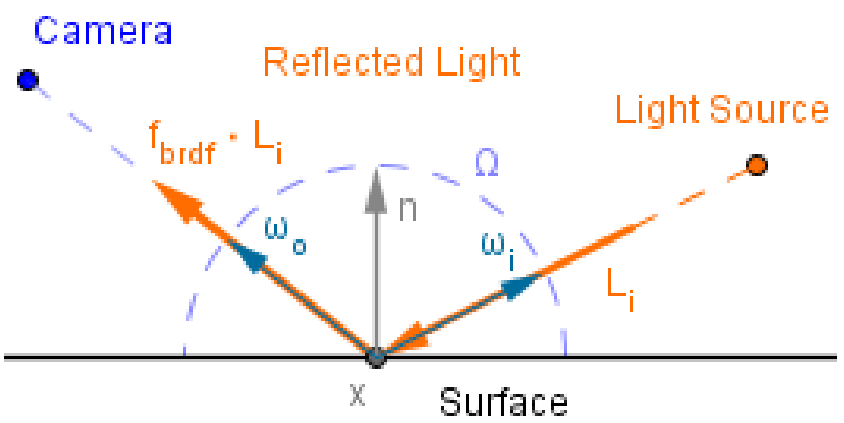
The Rendering Equation

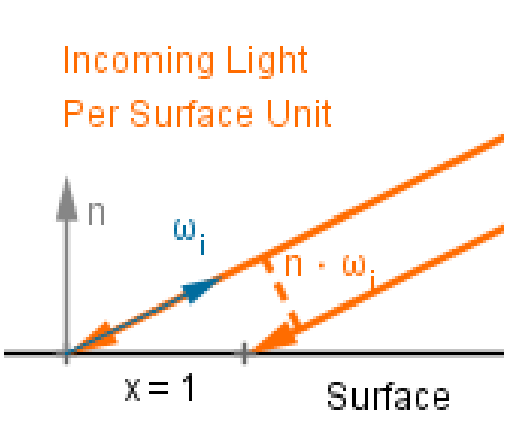
- Mathematical formulation for the visible color (outgoing light) of the surface.



$$L_{out}(x,\omega_o) = L_{emit}(x,\omega_o) + \int_{\Omega} f_{brdf}(x,\omega_i,\omega_o)\cdot L_{in}(x,\omega_i)\cdot(\omega_i\cdot n)\,\mathrm{d}\,\omega_i$$







The Rendering Equation

$$L_{out}(\boldsymbol{x}, \omega_o) = \\ = L_{emit}(\boldsymbol{x}, \omega_o) + \int_{\Omega} f_{brdf}(\boldsymbol{x}, \omega_i, \omega_o) \cdot L_{in}(\boldsymbol{x}, \omega_i) \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$

- $\boldsymbol{x}$ – Surface point

The Rendering Equation

$$L_{out}(x, \boldsymbol{\omega_o}) = L_{emit}(x, \boldsymbol{\omega_o}) + \int_{\Omega} f_{brdf}(x, \omega_i, \boldsymbol{\omega_o}) \cdot L_{in}(x, \omega_i) \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$

- x – Surface point
- $\boldsymbol{\omega_o}$ – Viewing (outgoing light) angle / vector

The Rendering Equation

$$L_{out}(x, \omega_o) = L_{emit}(x, \omega_o) + \int_{\Omega} f_{brdf}(x, \boldsymbol{\omega_i}, \omega_o) \cdot L_{in}(x, \boldsymbol{\omega_i}) \cdot (\boldsymbol{\omega_i} \cdot n) \, \mathrm{d}\omega_i$$

- x – Surface point
- ω_o – Viewing (outgoing light) angle / vector
- $\boldsymbol{\omega_i}$ – Incoming light angle / vector

The Rendering Equation

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- x – Surface point
- ω_o – Viewing (outgoing light) angle / vector
- ω_i – Incoming light angle / vector
- L_{out} – Amount of outgoing light

The Rendering Equation

$$L_{out}(x, \omega_o) = L_{emit}(x, \omega_o) + \int_{\Omega} f_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i) \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$

- x – Surface point
- ω_o – Viewing (outgoing light) angle / vector
- ω_i – Incoming light angle / vector
- L_{out} – Amount of outgoing light
- L_{emit} – Amount of emitted light

The Rendering Equation

$$L_{out}(x, \omega_o) = L_{emit}(x, \omega_o) + \int_{\Omega} \boldsymbol{f}_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i) \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$

- x – Surface point
- ω_o – Viewing (outgoing light) angle / vector
- ω_i – Incoming light angle / vector
- L_{out} – Amount of outgoing light
- L_{emit} – Amount of emitted light
- $\boldsymbol{f}_{brdf}$ – Amount of reflected light (BRDF)

Bidirectional Reflectance Distribution Function

The Rendering Equation

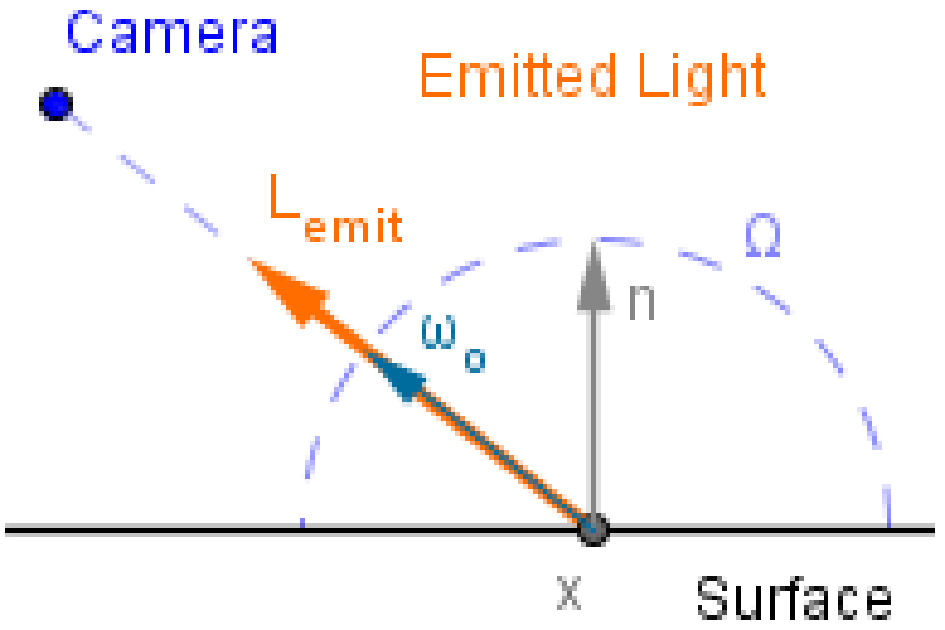
$$L_{out}(x, \omega_o) = L_{emit}(x, \omega_o) + \int_{\Omega} f_{brdf}(x, \omega_i, \omega_o) \cdot \boldsymbol{L_{in}(x, \omega_i) \cdot (\omega_i \cdot n)} \, \mathrm{d}\omega_i$$

- x – Surface point
- ω_o – Viewing (outgoing light) angle / vector
- ω_i – Incoming light angle / vector
- L_{out} – Amount of outgoing light
- L_{emit} – Amount of emitted light
- f_{brdf} – Amount of reflected light (BRDF)
- $\boldsymbol{L_{in} \cdot (\omega_i \cdot n)}$ – Amount of incoming light per direction

The Rendering Equation

- Some surfaces can emit light on their own (light sources *eg* car headlights, fluorescent materials)

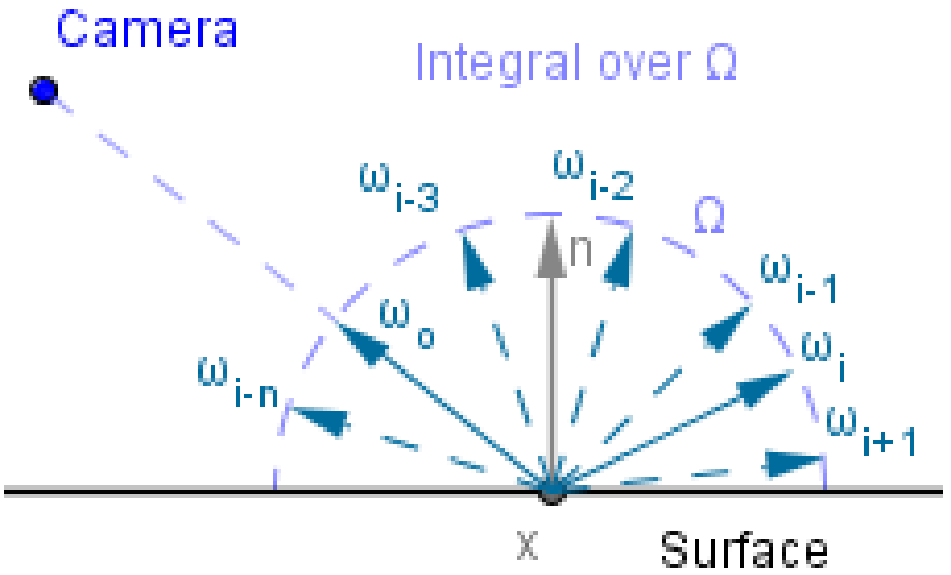
$$L_{out}(x, \omega_o) =$$
$$= \boldsymbol{L_{emit}(x, \omega_o)} + \int_{\Omega} f_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i) \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$



The Rendering Equation

- Outgoing light depends on the light coming in from all directions. Integrate over the hemisphere.

$$L_{out}(x, \omega_o) = = L_{emit}(x, \omega_o) + \int_{\Omega} f_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i) \cdot (\omega_i \cdot n) \, d\omega_i$$

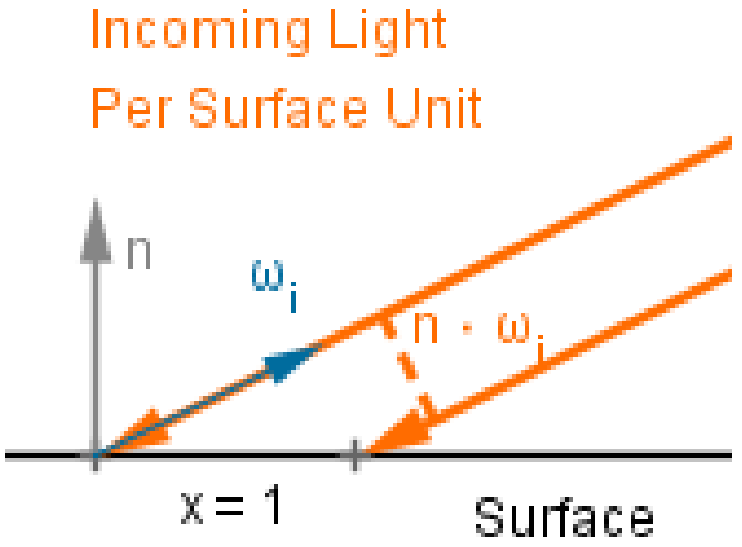


The Rendering Equation

- Light reaching the surface unit is proportional to

$$\cos\left(\angle\left(\omega_i, n\right)\right) = \left(\omega_i \cdot n\right)$$

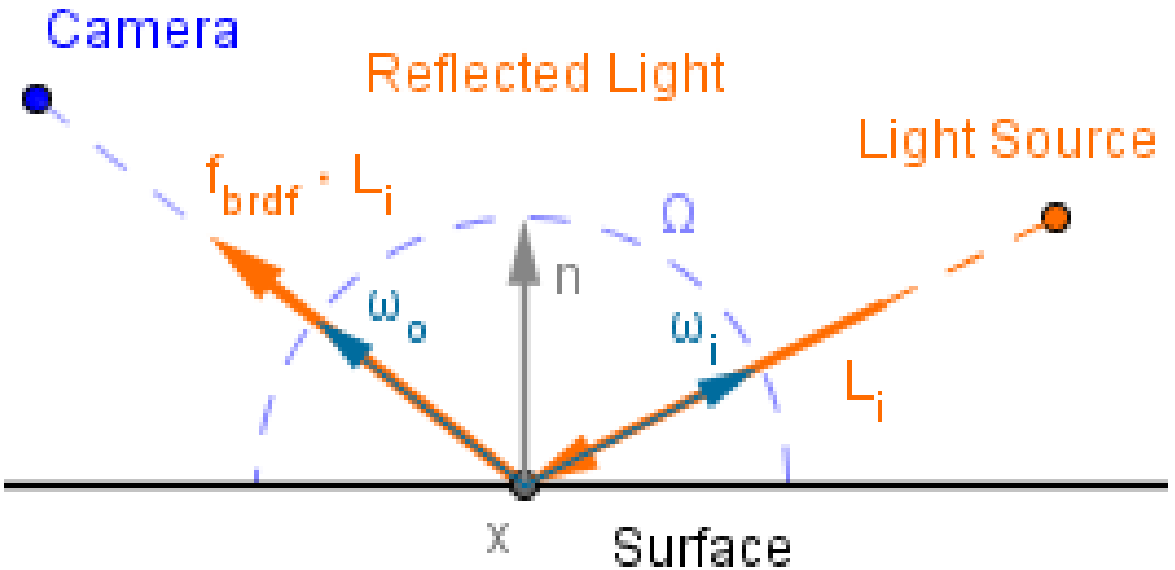
$$L_{out}(x, \omega_o) = \\ = L_{emit}(x, \omega_o) + \int_{\Omega} f_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i) \cdot \left(\boldsymbol{\omega_i \cdot n}\right) \mathrm{d}\,\omega_i$$



The Rendering Equation

- Reflected light depends on the incoming light L_{in} and the material properties represented by f_{brdf}
- BRDF – Bidirectional Reflectance Distribution Function

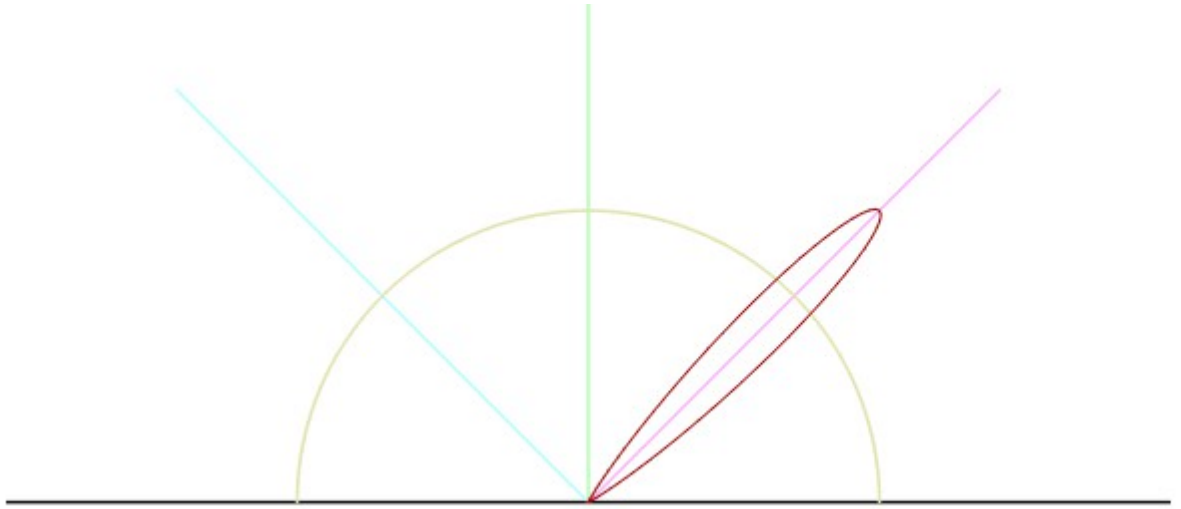
$$L_{out}(x, \omega_o) = L_{emit}(x, \omega_o) + \int_{\Omega} \boldsymbol{f_{brdf}(x, \omega_i, \omega_o) \cdot L_{in}(x, \omega_i)} \cdot (\omega_i \cdot n) \, \mathrm{d}\omega_i$$

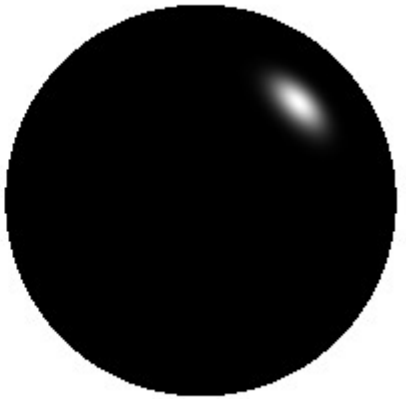


The Rendering Equation

- BRDF for Phong would be:

$$f_{brdf} = M_{diffuse} + M_{specular} \cdot (v \cdot r)^{shininess}$$





Disney's BRDF Explorer

The Rendering Equation

- BRDF for Phong would be:

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- Notice, that we are also multiplying the specular term with $\omega_i \cdot n$.

The Rendering Equation

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- The specular highlight intensity also depends on the angle light is reaching the surface.

The Rendering Equation

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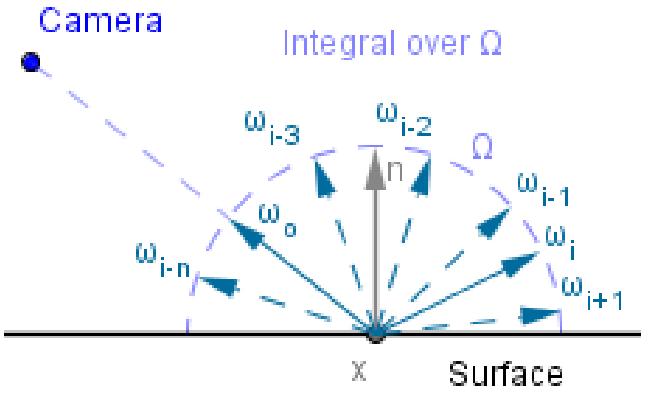
- Notice, that we are also multiplying the specular term with $\omega_i \cdot n$.
- The specular highlight intensity also depends on the angle light is reaching the surface.
- Alternatively:

$$f_{brdf} = M_{diffuse} + \frac{M_{specular} \cdot (v \cdot r)^{shininess}}{n \cdot l}$$

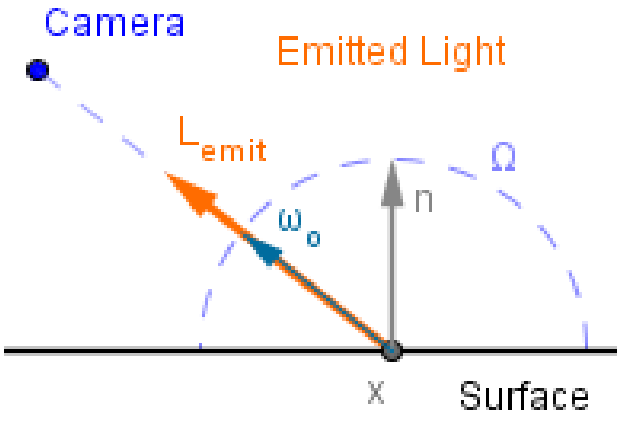
See this in the Disney's BRDF Explorer

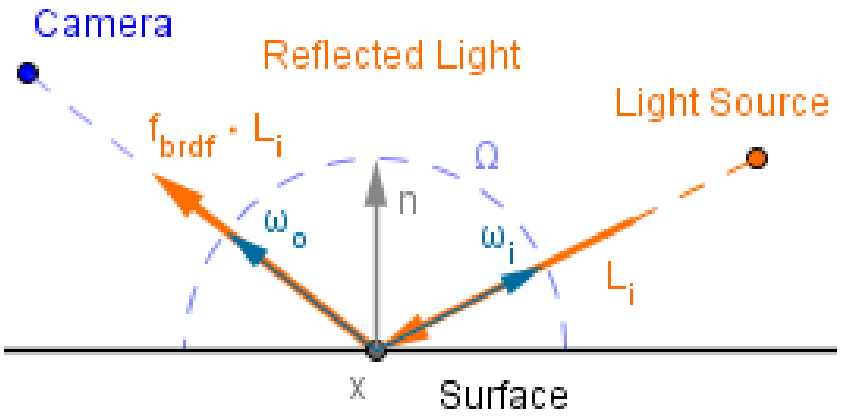
The Rendering Equation

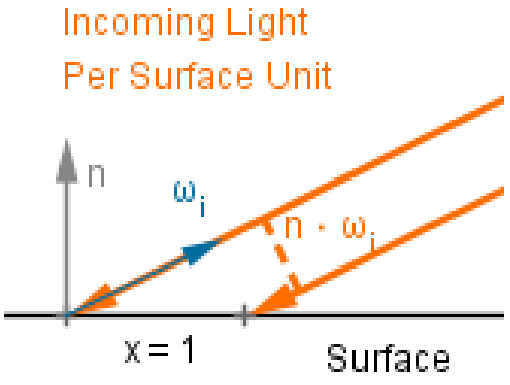
- Questions?



$$L_{out}(x,\omega_o) = L_{emit}(x,\omega_o) + \int_{\Omega} f_{brdf}(x,\omega_i,\omega_o)\cdot L_{in}(x,\omega_i)\cdot(\omega_i\cdot n)\,\mathrm{d}\,\omega_i$$

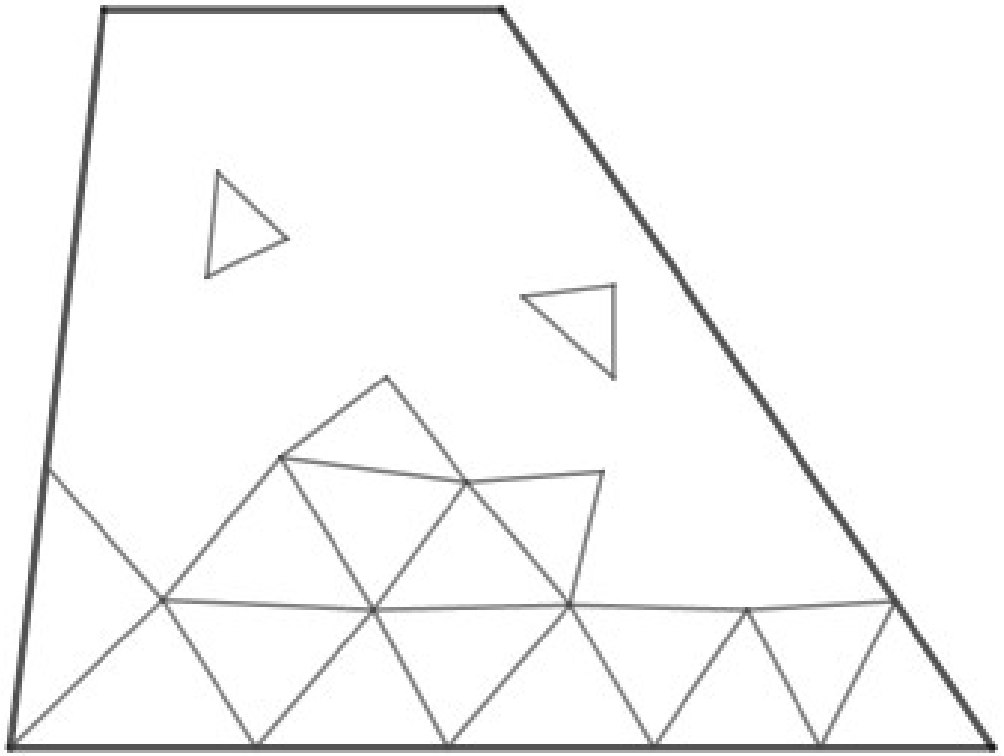






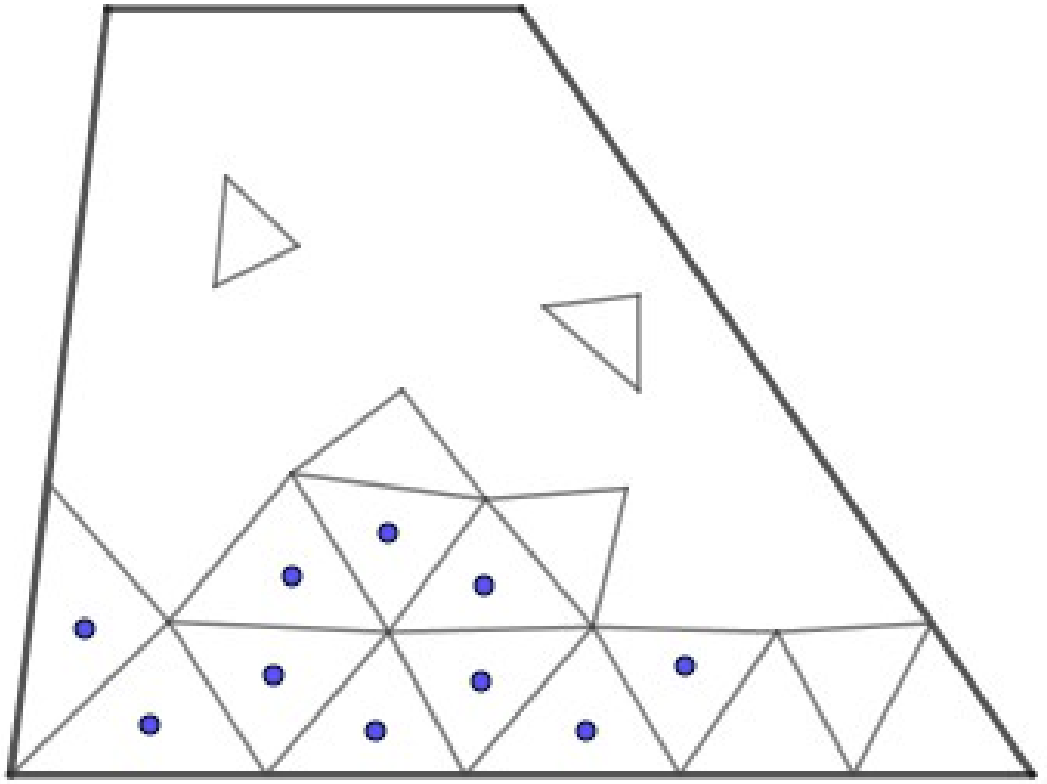
Radiosity

- Divides our geometry into (small) patches.



Radiosity

- Divides our geometry into (small) patches.
- Approximates the rendering equation by replacing the integral with a finite sum.



Radiosity

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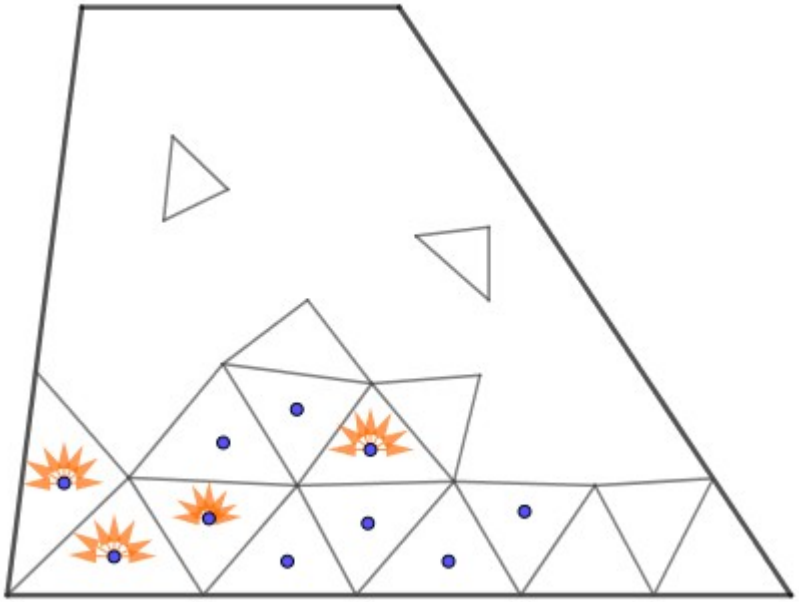

The BRDF is constant

Radiosity

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$$L_{out}(x_i,\omega_i)$$

Another patch x_i

Radiosity

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$\rho(x)$

What kind of function is this?

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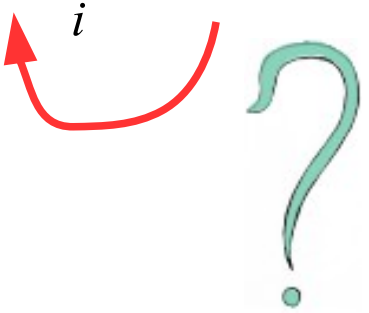

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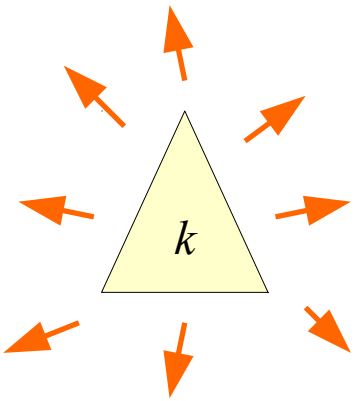
- For each patch k: $E_k+\rho_k\sum_j F_{k,j}\cdot L_j$



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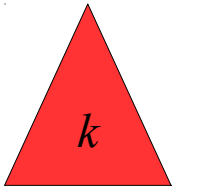

$\rho_k=[1,0,0]$

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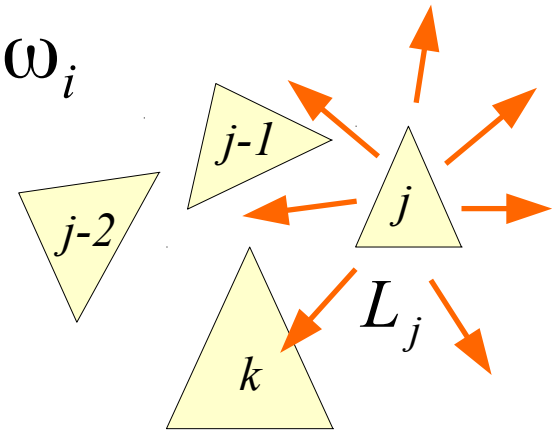

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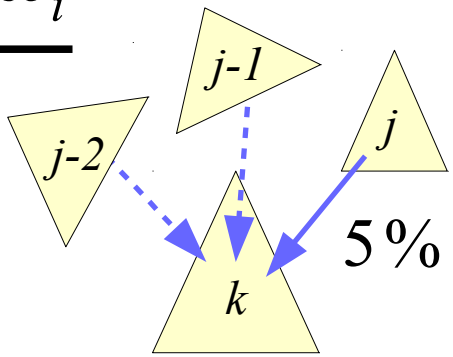

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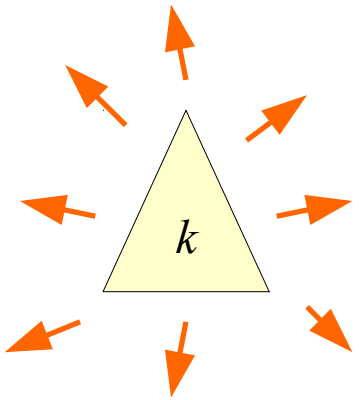
Radiosity

- For each patch k:

$$L_k = E_k + \rho_k \sum_j F_{k,j} \cdot L_j$$

How much this patch emits light



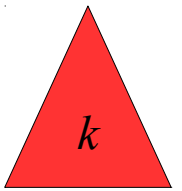
Radiosity

- For each patch k:

$$L_k = E_k + \rho_k \sum_j F_{k,j} \cdot L_j$$

How much this patch emits light

How much this patch reflects light (its color)



$\rho_k = [1, 0, 0]$

Radiosity

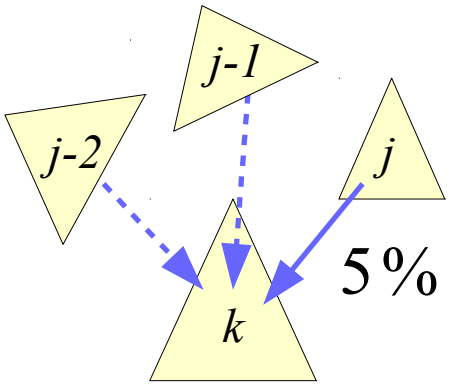
- For each patch k:

$$L_k = E_k + \rho_k \sum_j F_{k,j} \cdot L_j$$

How much this patch emits light

How much this patch reflects light (its color)

View factor
How much this patch k receives light from patch j

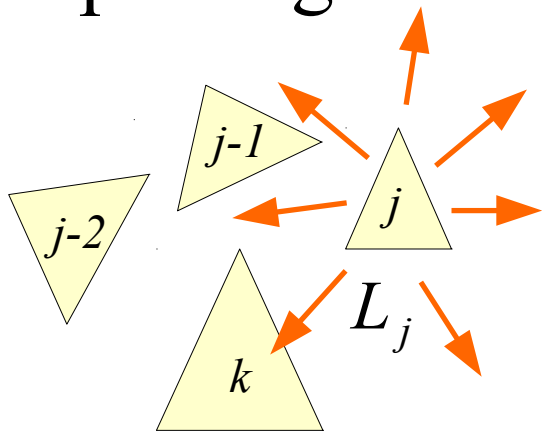


Radiosity

- For each patch k:

$$L_k = E_k + \rho_k \sum_j F_{k,j} \cdot L_j$$

How much patch j outputs light



How much this patch emits light

How much this patch reflects light (its color)

View factor
How much this patch k receives light from patch j

Radiosity

- For each patch k:

$$L_k = E_k + \rho_k \sum_j F_{k,j} \cdot L_j$$

$$F = \begin{pmatrix} 0 & F_{0,1} & F_{0,2} & \dots & F_{0,k} \\ F_{1,0} & 0 & F_{1,2} & \dots & F_{1,k} \\ \dots & \dots & \dots & \dots & \dots \\ F_{k,0} & F_{k,1} & F_{k,2} & \dots & 0 \end{pmatrix}$$

View factors in each row need to sum up to 1.
We can not have more than 100% energy entering / exiting a patch.
We also assume an enclosed system where energy is conserved.

Radiosity

- Vector equation for the entire scene

$$L = E + \rho F \cdot L$$

$$\begin{pmatrix} L_0 \\ L_1 \\ \dots \\ L_k \end{pmatrix} = \begin{pmatrix} E_0 \\ E_1 \\ \dots \\ E_k \end{pmatrix} + \rho \begin{pmatrix} 0 & F_{0,1} & F_{0,2} & \dots & F_{0,k} \\ F_{1,0} & 0 & F_{1,2} & \dots & F_{1,k} \\ \dots & \dots & \dots & \dots & \dots \\ F_{k,0} & F_{k,1} & F_{k,2} & \dots & 0 \end{pmatrix} \cdot \begin{pmatrix} L_0 \\ L_1 \\ \dots \\ L_k \end{pmatrix}$$

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Diagonal matrix with $\rho_0, \rho_1, \dots, \rho_k$

Radiosity

- Vector equation for the entire scene

$$L = E + \rho F \cdot L$$

$$-E = \rho F \cdot L - L$$

Radiosity

- Vector equation for the entire scene

$$L = E + \rho F \cdot L$$

$$-E = \rho F \cdot L - L$$

$$-E = (\rho F - I) L$$

Radiosity

- Vector equation for the entire scene

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$$-E = (\rho F - I) L$$

$$E = (I - \rho F) L$$

Radiosity

- Vector equation for the entire scene

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$$E = (I - \rho F) L$$

$$L = (I - \rho F)^{-1} E$$

Radiosity

- Vector equation for the entire scene

$$L = E + \rho F \cdot L$$
$$-E = \rho F \cdot L - L$$
$$-E = (\rho F - I) L$$
$$E = (I - \rho F) L$$
$$L = (I - \rho F)^{-1} E$$

- Unfortunately, finding the solution this way is $O(k^3)$

k – Number of patches...

http://mathforum.org/library/drmath/view/51908.html

Radiosity

- Jacobi iteration method solves $Ax = b$, when A is strictly row diagonally dominant.

Radiosity

- Jacobi iteration method solves $Ax = b$, when A is **strictly row diagonally dominant**.

Diagonal element has greater absolute value, then the sum of absolute values of other elements in a row.

$$|a_{i,i}| > \sum_{i \neq j} |a_{i,j}|$$

Radiosity

- Jacobi iteration method solves $Ax = b$, when A is strictly row diagonally dominant.

$$\left(I - \rho F\right) L = E$$

Radiosity

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$$(I - \rho F) L = E$$

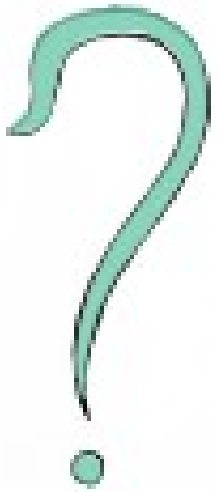
$$\underbrace{\left(I - \rho \begin{pmatrix} 0 & F_{0,1} & F_{0,2} & \cdots & F_{0,k} \\ F_{1,0} & 0 & F_{1,2} & \cdots & F_{1,k} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ F_{k,0} & F_{k,1} & F_{k,2} & \cdots & 0 \end{pmatrix} \right)}_{A} \cdot \underbrace{\begin{pmatrix} L_0 \\ L_1 \\ \cdots \\ L_k \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} E_0 \\ E_1 \\ \cdots \\ E_k \end{pmatrix}}_{b}$$

Radiosity

- Jacobi iteration method solves $Ax = b$, when A is strictly row diagonally dominant.

$$(I - \rho F) L = E$$

- Is $(I - \rho F)$ strictly diagonally dominant? When?



Radiosity

- Jacobi iteration method finds an approximation:

$$L^{(r+1)} = D^{-1}\left(E - R \cdot L^{r}\right)$$

Radiosity

- Jacobi iteration method finds an approximation:

$$L^{(r+1)} = D^{-1}\left(E - R \cdot L^{r}\right)$$

D – diagonal of $\left(I - \rho F\right)$

What is that value?

Radiosity

- Jacobi iteration method finds an approximation:

$$L^{(r+1)} = D^{-1}\left(E - R \cdot L^{r}\right)$$

$$R = \left(I - \rho F\right) - D = -\rho F$$

The „remainder“ of A (A minus the diagonal)

Radiosity

- Jacobi iteration method finds an approximation:

$$L^{(r+1)} = D^{-1}\left(E - R \cdot L^{r}\right)$$

$$L^{(r+1)} = E - R \cdot L^{r}$$

$$L^{(r+1)} = E + \rho F \cdot L^{r}$$

This is similar to, what we started with...

Radiosity

- Jacobi iteration method finds an approximation:

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L^{r} – r-th approximation

L^{0} – initial random guess

This is similar to, what we started with...

Radiosity

- Difficult part is to **find the view factors** for all the patches.

$O(k^2)$

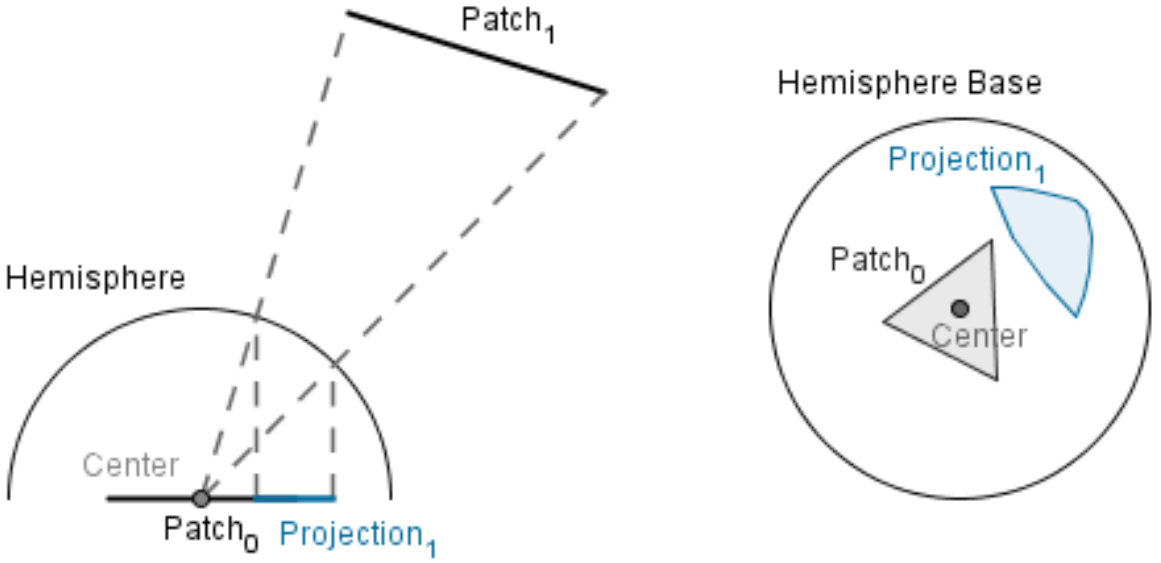
Naive way

$O(k \cdot \log k)$

With BSP tree

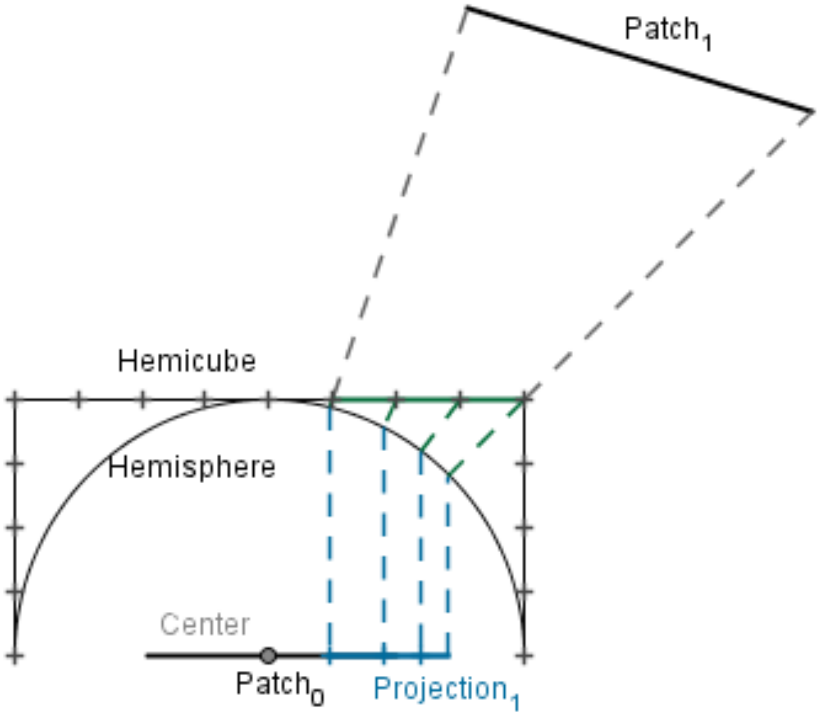
Radiosity

- Difficult part is to **find the view factors** for all the patches. One way to do it, is to:
  1) Create an unit hemisphere around the patch
  2) Project another patch onto the hemisphere
  3) Then project that onto the base of the hemisphere
  4) Ratio between the area of the last projection and the area of the circular base, is the view factor.



Radiosity

- Because finding the actual hemisphere projection for each patch is again complicated, we can create a hemicube.
- We create a rectangular grid on the hemicube and pre-calculate the hemisphere projections of each cell.
- Then we project other patches onto the cube, and see what cells they cover.



Global Illumination

- Different combinations of the algorithms.
- Many algorithms for real time approximations.

- **Voxel Octree Cone Tracing** by Nvidia, 2011
  - Real-time application
  - https://research.nvidia.com/publication/interactive-indirect-illumination-using-voxel-cone-tracing
- **Lightmass – Unreal Engine 4**:
  - Photon Mapping approach
  - https://docs.unrealengine.com/latest/INT/Engine/Rendering/LightingAndShadows/Lightmass/index.html

BRDF

- **Disney BRDF explorer**
  - See different BRDF-s (incl. Lambert and Phong).
  - http://www.disneyanimation.com/technology/brdf.html
- **An Overview of BRDF Models** (2012)
  - Great survey of different BRDF-s
  - https://digibug.ugr.es/bitstream/handle/10481/19751/rmontes_LSI-2012-001TR.pdf
- Microfacet BRDF-s by Simon's Tech Blog
  - Includes interactive demos
  - http://simonstechblog.blogspot.com/2011/12/microfacet-brdf.html

What ideas you had today?

What more would you like to know?

Next time: Shadows, Conclusion!