Computer Graphics
MTAT.03.015

Raimond Tunnel

Study IT in .ee
The Road So Far...
Curves

• Line interpolates between 2 points.
Curves

- Line interpolates between 2 points.
- Mathematically there are higher polynomials to interpolate between more points.
Curves

- Line interpolates between 2 points.
- Mathematically there are higher polynomials to interpolate between more points.
- How many points you need, to construct a $n$-th degree polynomial through it?
Curves

How to find that parabola given the 3 points?

$2x + 1y = 5$

$y = x^2 - 5x + 7$
Curves

• Constructing a parabola through 3 points:

\[ f(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0 \]
Curves

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• We know: \( f(1) = 3, \ f(2) = 1, \ f(3) = 1 \)
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• 3 unknowns, 3 constraints, we can solve it.

• http://www.wolframalpha.com/input/?i=a*1+%2B+b*1+%2B+c+%2B+d+e+f+g+h+i+j+k+l+m+n+o+p+q+r+s+t+u+v+w+x+y+z

Curves

• What choices we have with 4 points?

One additional point meant another line, could we have 2 parabolas here?
Curves

- Constraints do not have to be on the function.
Curves

- Constraints do not have to be on the function.
- They can also be on the derivative of it.

\[ g(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0 \]
\[ g'(x) = ? \]
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- Constraints:

\[ g(3) = 1 \]
\[ g(5) = 2 \]
\[ g'(3) = f'(3) = ? \]
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Curves

\[ y = x^2 - 5x + 7 \]

\[ y = -0.25x^2 + 2.5x - 4.25 \]
Smoothness

• What if we have 5 points and we put two parabolas through them without accounting for the derivative?
Smoothness

- That spline is not $C^1$ smooth.
Smoothness (continuity)

- **Spline** – one or many connected curves.
Smoothness (continuity)

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- \( C^n \) **smoothness** – the \( n \)-th derivative is continuous everywhere along the object and the object is also \( C^{n-1} \) smooth.

The differentiability class
Smoothness (continuity)

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- For **parametric curves**, we can also talk about:
  - **$G^n$ smoothness** (geometric smoothness) – the $n$-th derivative can have sudden jumps in magnitude, but not the direction.
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- For **parametric curves**, we can also talk about:
  - **$G^n$ smoothness** (geometric smoothness) – the $n$-th derivative can have sudden jumps in magnitude, but not the direction. And the object is of $G^{n-1}$
Smoothness (continuity)

• Different levels of smoothness:
  • $C^0$ – Curve itself is continuous
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- Often times $C^1$ or $C^2$ smooth curves are enough in computer graphics.
- If we put quadratic curves together, so that the spline is $C^1$ smooth, how to get $C^2$ smoothness?

Find the second derivatives of our previous example...
Parametric Curves

- Implicit form: \( f(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0 \)
  - Good for testing points in a curve
  - Finding collisions
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For your regular mathematical quadratic fun.
Parametric Curves

- Implicit form: \( f(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0 \)
  - Good for testing points in a curve
  - Finding collisions

- Parametric form: \( g(t) = (t + x_0, a_2 \cdot t^2 + y_0) = (x, y) \)

\[
x_0 = \frac{-a_1}{2 \cdot a_2}, \quad y_0 = f(x_0)
\]

- Good for \textit{generating points on the curve}

Parametric form of a quadratic polynomial: http://www.nabla.hr/PC-ParametricEqu2.htm
Parametric Curves

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  - Good for testing points in a curve
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- Parametric form: $g(t) = (t + x_0, a_2 \cdot t^2 + y_0) = (x, y)$
  \[ x_0 = \frac{-a_1}{2 \cdot a_2}, \quad y_0 = f(x_0) \]
  - Good for generating points on the curve
  - **What other parametric equations you know?**
Parametric Curve Construction

- We want to find the **vector coefficients** \( a_i \) for a function of \( t \) (time), where \( t \in [0..1] \).

  Quadratic: \[
  \text{curve}(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2
  \]

  Cubic: \[
  \text{curve}(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3
  \]
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- We need to have constraints. For example, the curve must **interpolate** a number of 2D points.
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- We need to have constraints. For example, the curve must **interpolate** a number of 2D points.

- How many points we need?
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- For a quadratic curve in 2D we need 3 points. Each 2D point gives 2 1D constraints.
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- What about in 3D? Cubic?
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\begin{align*}
\text{curve}(0) &= (1, 3) = p_0 \\
\text{curve}(0.5) &= (2, 1) = p_1 \\
\text{curve}(1) &= (3, 1) = p_2
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\]

- Usually the system of constraints is written in a constraint matrix.
Parametric Curve Construction

- Constraint matrix

\[
\begin{pmatrix}
1 & 0 & 0 \\
1 & 0.5 & 0.25 \\
1 & 1 & 1 \\
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
a_2 \\
\end{pmatrix}
= 
\begin{pmatrix}
p_0 \\
p_1 \\
p_2 \\
\end{pmatrix}
\]
Parametric Curve Construction

- **Constraint matrix**

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In short: \( C \cdot a = p \)

- Write out the equations to see, that this is exactly what we did before with the implicit eq.
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- Only now the \( a_i \) and \( p_i \) are vectors.
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In short: \( C \cdot a = p \)

How to find \( a \)?

- Write out the equations to see, that this is exactly what we did before.

- Only now the \( a_i \) and \( p_i \) are vectors.
Parametric Curve Construction

• We can find $a = C^{-1} p$, the inverse constraint matrix $C^{-1}$ is often denoted $B$ and called the basis / blending matrix.
Parametric Curve Construction

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In our example:

$B = \begin{pmatrix}
1 & 0 & 0 \\
-3 & 4 & -1 \\
2 & -4 & 2
\end{pmatrix}$

Inverse (and other stuff) matrix calculator: http://www.bluebit.gr/matrix-calculator/
Parametric Curve Construction

• We can find $a = C^{-1} p$, the inverse constraint matrix $C^{-1}$ is often denoted $B$ and called the basis / blending matrix.

• Now we know the coefficients in:

$$\text{curve}(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2$$

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a_0 = b_{0,0} p_0 + b_{0,1} p_1 + b_{0,2} p_2
\]

\[
a_1 = b_{1,0} p_0 + b_{1,1} p_1 + b_{1,2} p_2
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\[
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Parametric Curve Construction

Let us look at the entire curve:

\[
\text{curve}(t) = b_{0,0}p_0 + b_{0,1}p_1 + b_{0,2}p_2 + b_{1,0}p_0t + b_{1,1}p_1t + b_{1,2}p_2t + b_{2,0}p_0t^2 + b_{2,1}p_1t^2 + b_{2,2}p_2t^2
\]
Parametric Curve Construction

• Let us look at the entire curve:

\[ \text{curve}(t) = \]
\[ = b_{0,0} p_0 + b_{0,1} p_1 + b_{0,2} p_2 + \]
\[ + b_{1,0} p_0 t + b_{1,1} p_1 t + b_{1,2} p_2 t + \]
\[ + b_{2,0} p_0 t^2 + b_{2,1} p_1 t^2 + b_{2,2} p_2 t^2 \]

• We can rewrite it as:

\[ \text{curve}(t) = b_0(t) \cdot p_0 + b_1(t) \cdot p_1 + b_2(t) \cdot p_2 \]
Parametric Curve Construction

- Let us look at the entire curve:

\[
\text{curve}(t) = \begin{align*}
&= b_{0,0} p_0 + b_{0,1} p_1 + b_{0,2} p_2 + \\
&+ b_{1,0} p_0 t + b_{1,1} p_1 t + b_{1,2} p_2 t + \\
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\text{curve}(t) = b_0(t) \cdot p_0 + b_1(t) \cdot p_1 + b_2(t) \cdot p_2
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\[
b_i(t) = b_{0,i} + b_{1,i} \cdot t + b_{2,i} \cdot t^2
\]

Coefficients from one \((i\text{-th})\) column of the matrix \(B\).
Parametric Curve Construction

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b_i(t) = b_{0,i} + b_{1,i} \cdot t + b_{2,i} \cdot t^2
\]

The functions \( b_i \) are called basis / blending functions.
Parametric Curve Construction

- We have constructed a quadratic equation of time to interpolate our control points!
Parametric Curve Construction

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- Similar construction can be done for cubic equations and different other constraints (besides interpolation).
Parametric Curve Construction

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- Similar construction can be done for cubic equations and different other constraints (besides interpolation).

1) Pick a degree of the curve
2) Fix the parameters \((incl)\ control points\)
3) Create the constraint matrix \(C\)
4) Find the basis matrix \(B = C^{-1}\)
5) Read the blending functions from the basis matrix
Blending Functions

• Used to interpolate between the parameters.
Blending Functions

- Used to interpolate between the parameters.

- Here are the found blending functions for interpolating a quadratic curve between 3 points:

\[-x + 2 \cdot x^2\]

\[4 \cdot x - 4 \cdot x^2\]

\[1 - 3 \cdot x + 2 \cdot x^2\]
Blending Functions

● Used to interpolate between the parameters.
● Here are the found blending functions for interpolating a quadratic curve between 3 points:

\[-x + 2 \cdot x^2\]
\[4 \cdot x - 4 \cdot x^2\]
\[1 - 3 \cdot x + 2 \cdot x^2\]

● Different constraints give different functions
Cubic not Quadratic

- In computer graphics, we usually want to use cubic polynomials, not quadratics.
Cubic not Quadratic

- In computer graphics, we usually want to use cubic polynomials, not quadratics.
- Cubic polynomials provide us with 4 possible constraints.
Cubic not Quadratic

• In computer graphics, we usually want to use cubic polynomials, not quadratics.
• Cubic polynomials provide us with 4 possible constraints.
• Splines can achieve $C^2$ smoothness.
Hermite Spline

- The derivatives at the endpoints are parameters.
- Segments share the endpoints and derivatives.

\[
\begin{align*}
\text{curve}(0) &= p_0 \\
\text{curve}'(0) &= p_1 \\
\text{curve}(1) &= p_2 \\
\text{curve}'(1) &= p_3
\end{align*}
\]
Hermite Spline

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\end{align*}
\]
Catmull-Rom Spline

- We interpolate the $p_1$ and $p_2$.
- Derivatives are calculated using the other points.

\[
\text{curve}'(0) = 0.5 \cdot (p_2 - p_0) \\
\text{curve}(0) = p_1 \\
\text{curve}(1) = p_2 \\
\text{curve}'(1) = 0.5 \cdot (p_3 - p_1)
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- Only specify start and end derivatives, others are calculated.
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\text{curve}(0) &= p_1 \\
\text{curve}(1) &= p_2 \\
\text{curve}'(1) &= 0.5 \cdot (p_3 - p_1)
\end{align*}
\]

- Only specify start and end derivatives, others are calculated.
Bezder Curve

- Could be constructed using the constraints and finding the blending functions.
- Could also be constructed in a procedural way:
  - Subdivide the lines connecting the control points, into proportions $t$ and $(1-t)$.
  - Do it recursively until at last subdivision, which will give a point on the curve.
Bezier Curve

- That procedure is called De Casteljau's algorithm.
Bezíer Curve

- That procedure is called De Casteljau's algorithm.
- The corresponding blending functions are called Bernstein basis polynomials.

\[
\begin{align*}
    b_{0,0}(t) &= 1 \\
    b_{0,1}(t) &= 1 - t, \quad b_{1,1}(t) = t \\
    b_{0,2}(t) &= (1 - t)^2, \quad b_{1,2}(t) = 2 \cdot t \cdot (1 - t), \quad b_{2,2}(t) = t^2 \\
    b_{i,\text{degree}}(t) &= \binom{\text{degree}}{i} \cdot t^i \cdot (1 - t)^{\text{degree} - i}
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\]
Beziers Curve

- That procedure is called De Casteljau's algorithm.
- The corresponding blending functions are called Bernstein basis polynomials.

\[
b_{0,0}(t) = 1 \\
b_{0,1}(t) = 1 - t, \quad b_{1,1}(t) = t \\
b_{0,2}(t) = (1 - t)^2, \quad b_{1,2}(t) = 2 \cdot t \cdot (1 - t), \quad b_{2,2}(t) = t^2 \\
b_{i,\text{degree}}(t) = \binom{\text{degree}}{i} \cdot t^i \cdot (1 - t)^{\text{degree} - i}
\]
Beziers Curve

• Always inside the convex hull of the control points.
Bezier Curve

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- Affine invariance – affine transformations on the control points, transform the curve itself correctly too.
Bezner Curve

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Beziers Curve

- Always inside the convex hull of the control points.
- Affine invariance – affine transformations on the control points, transform the curve itself correctly too.
- Sufficiently smooth splines can be constructed (Stärk's construction, we will see in the practice)
- Very widely used (eg font rendering)
Cubic Splines

- When constructing cubic splines, only 2 of the following properties can be satisfied at once:
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  b) Spline interpolates the control points.
  
  c) Spline has local control (changes in control points do not generally affect the entire curve).
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  a) Spline is $C^2$ smooth.
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- Hermite and Catmull-Rom – are not $C^2$ smooth.
- Bezier – does not interpolate the control points.
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