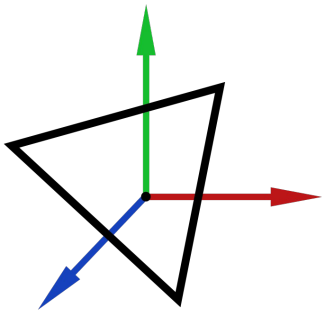
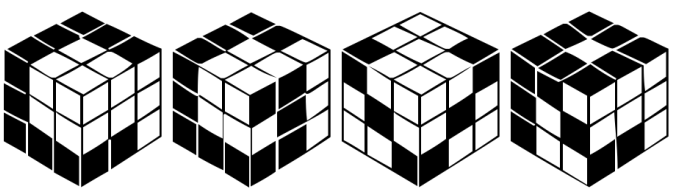


Computer Graphics

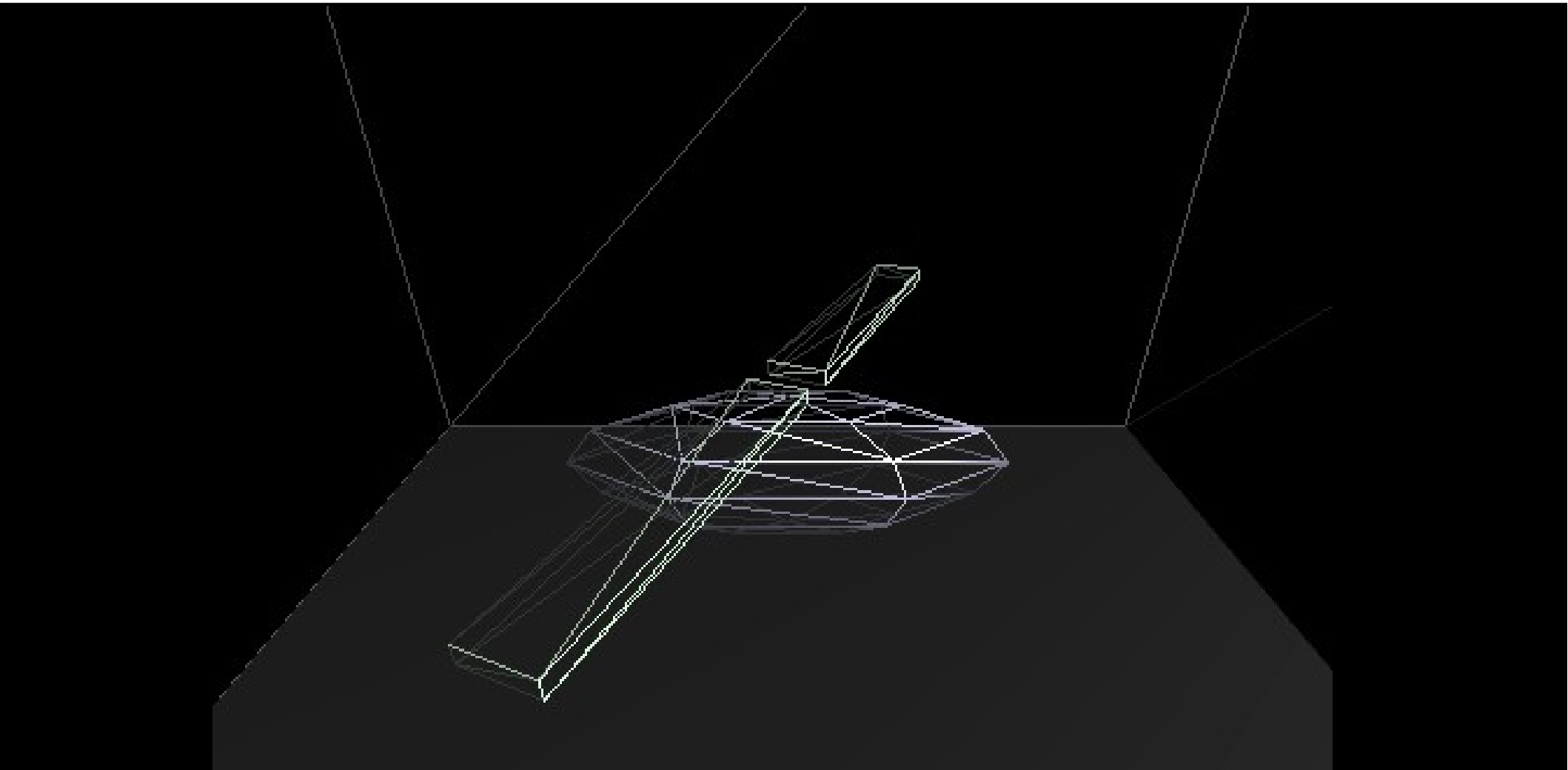
MTAT.03.015

Raimond Tunnel

UNIVERSITY OF TARTU
Institute of Computer Science

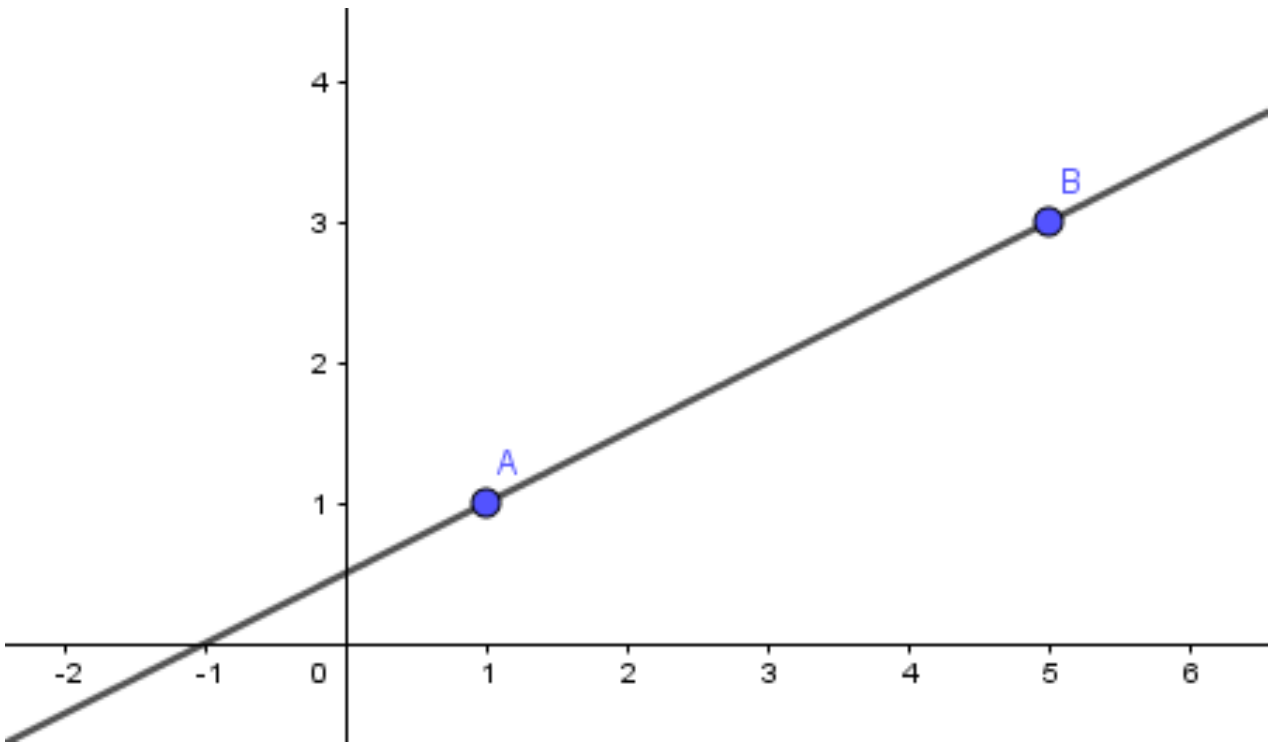


Previously



Curves

- Line interpolates between 2 points.



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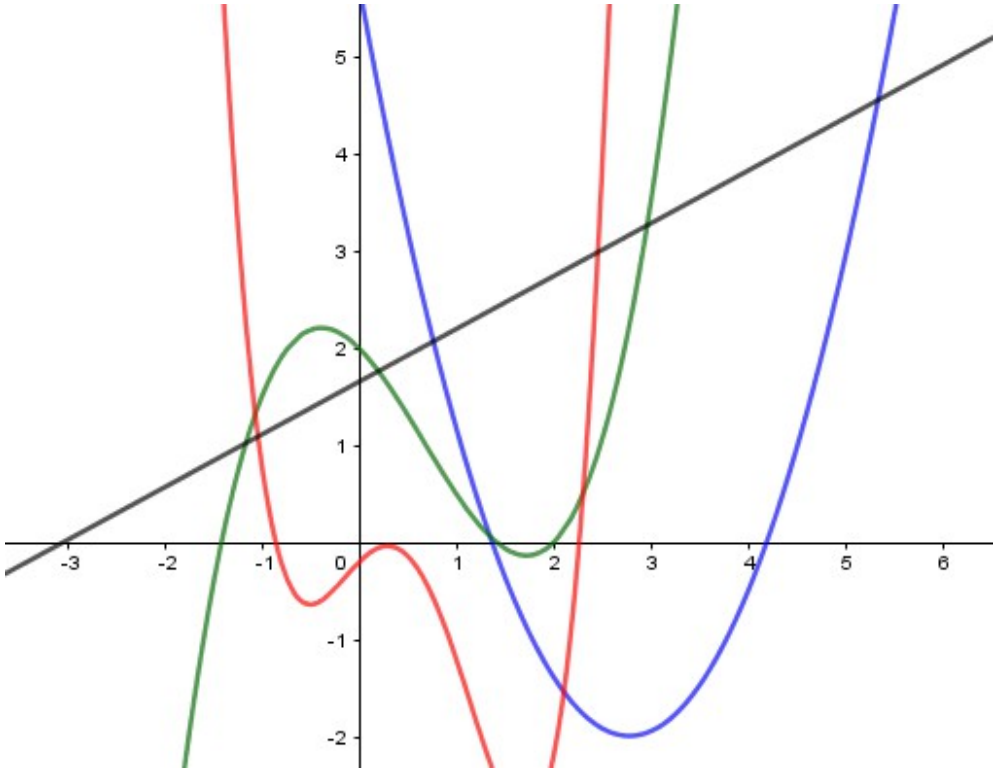
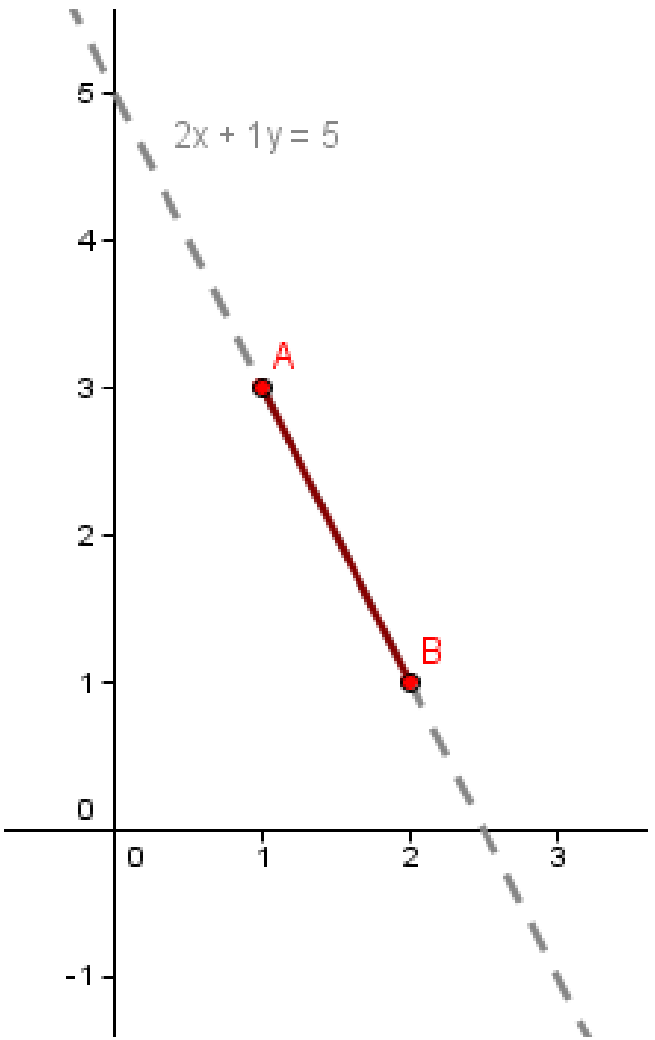
- Line interpolates between 2 points.
- Mathematically there are higher degree polynomials to interpolate more points.

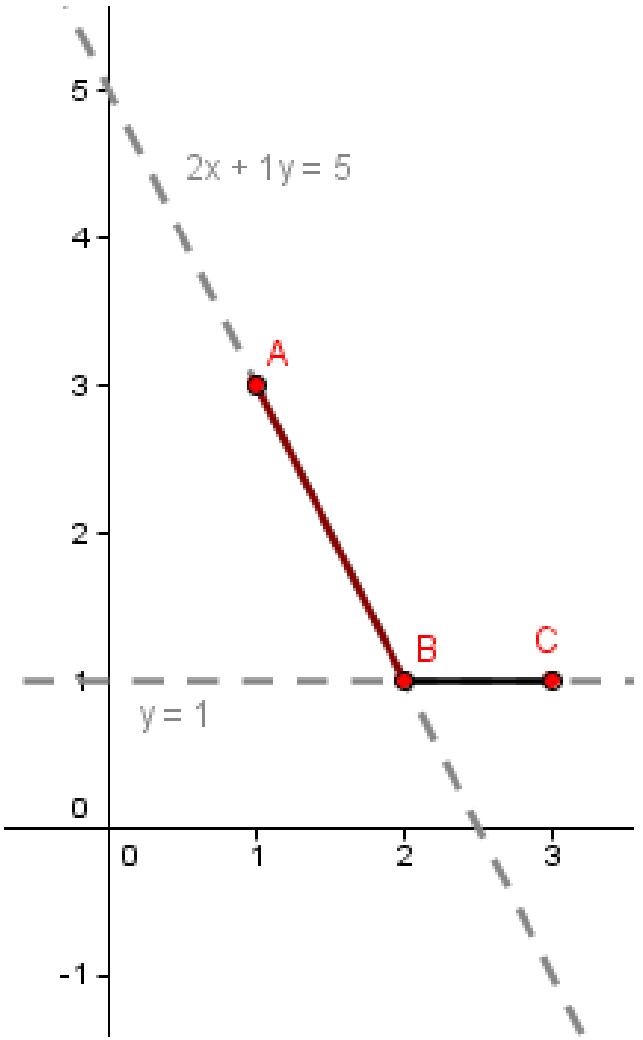


Curves

- Line interpolates between 2 points.
- Mathematically there are higher degree polynomials to interpolate more points
- How many points you need, to construct a n-th degree polynomial through it?

Curves


2x + 1y = 5
A
B
5
4
3
2
1
0
-1
0
1
2
3


2x + 1y = 5
A
B
C
y = 1
5
4
3
2
1
0
-1
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1
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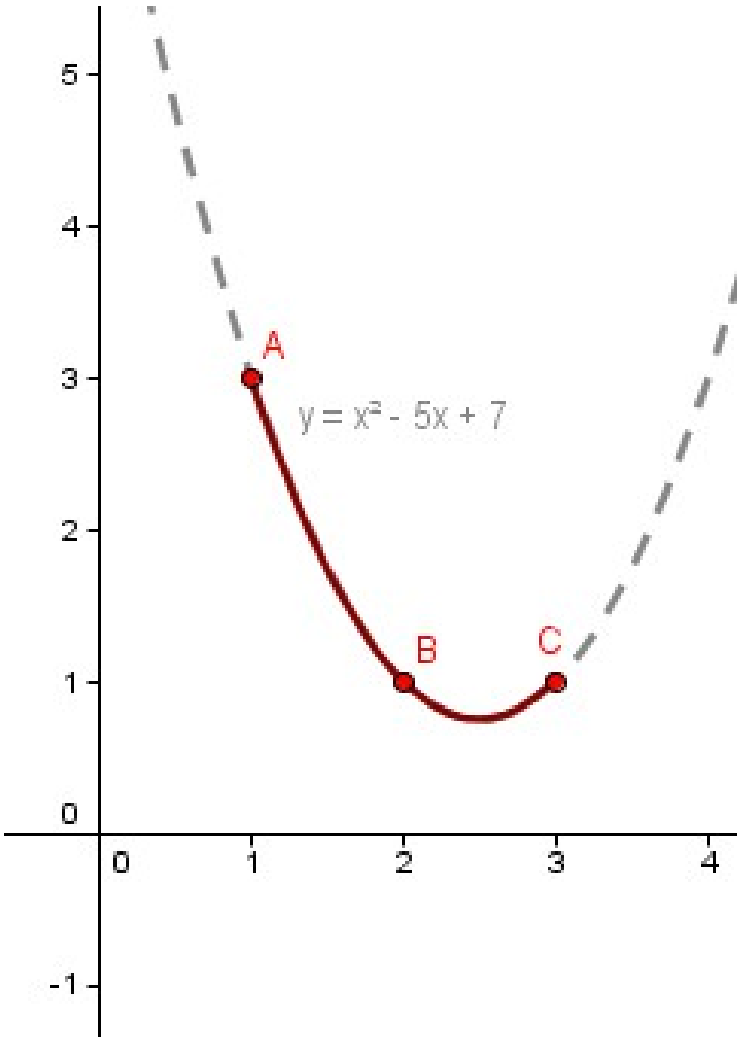
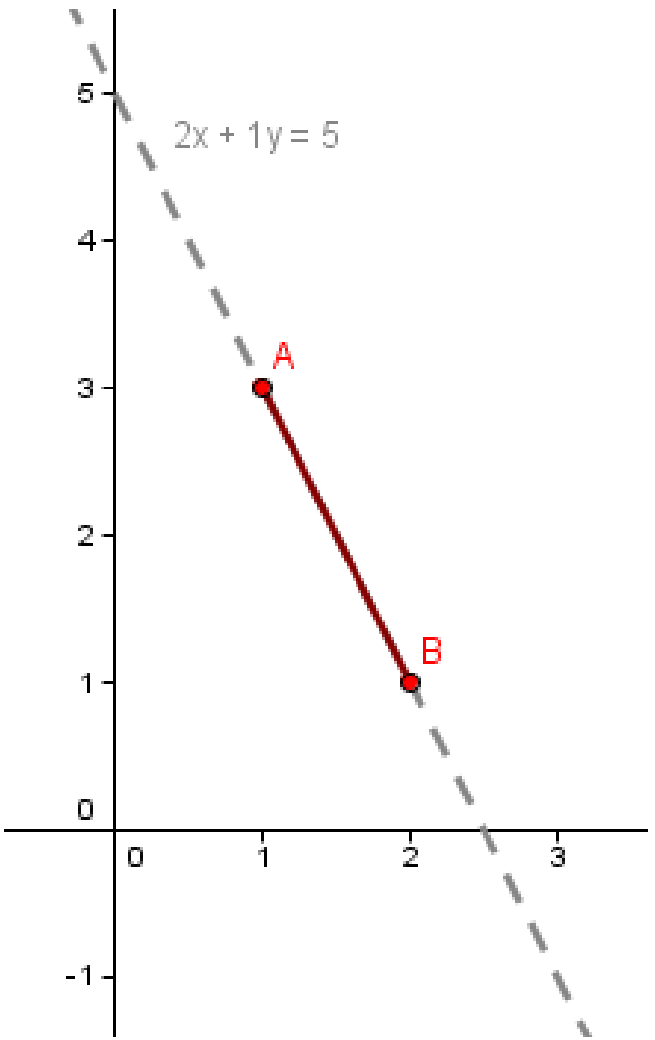
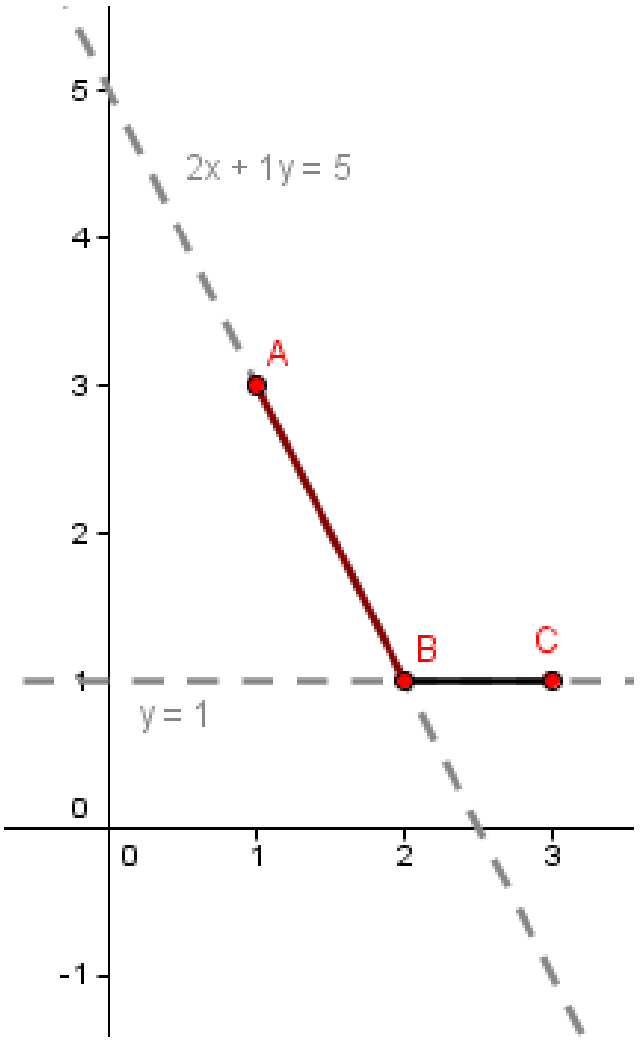
3

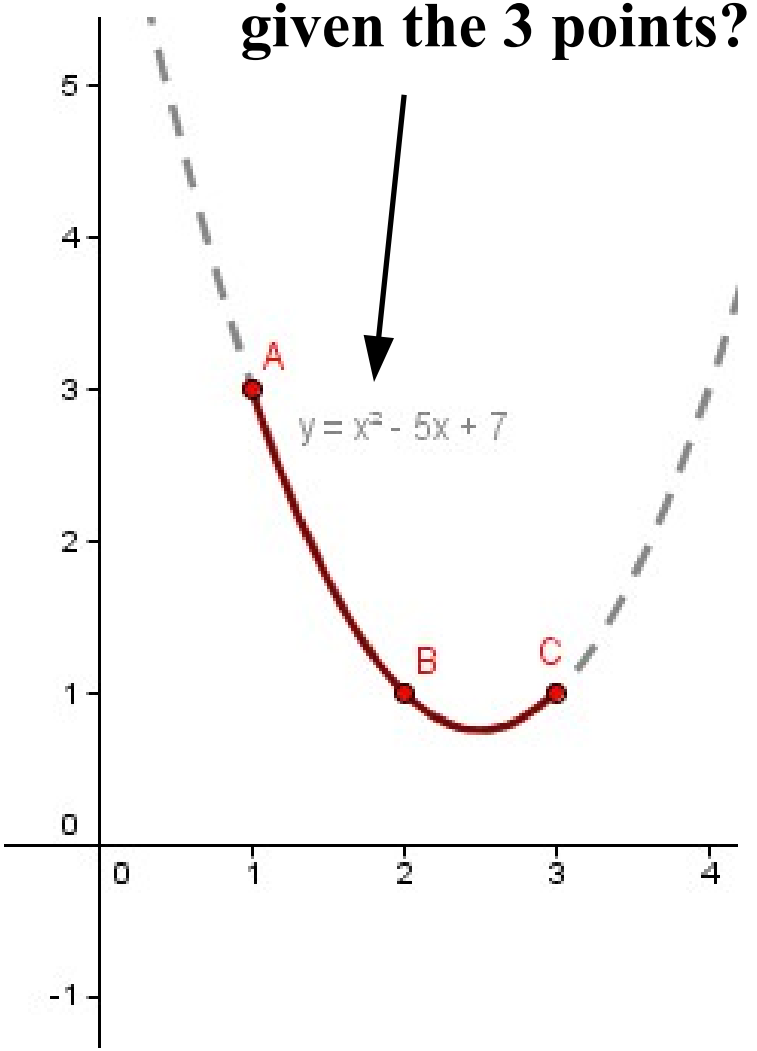

A
y = x² - 5x + 7
B
C
5
4
3
2
1
0
-1
0
1
2
3
4

Curves





How to find that parabola given the 3 points?



Curves

- Constructing a parabola through 3 points:

$$f(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0$$

Just like we found a line equation in the first lectures

Curves

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- We are looking for: a_2, a_1, a_0.

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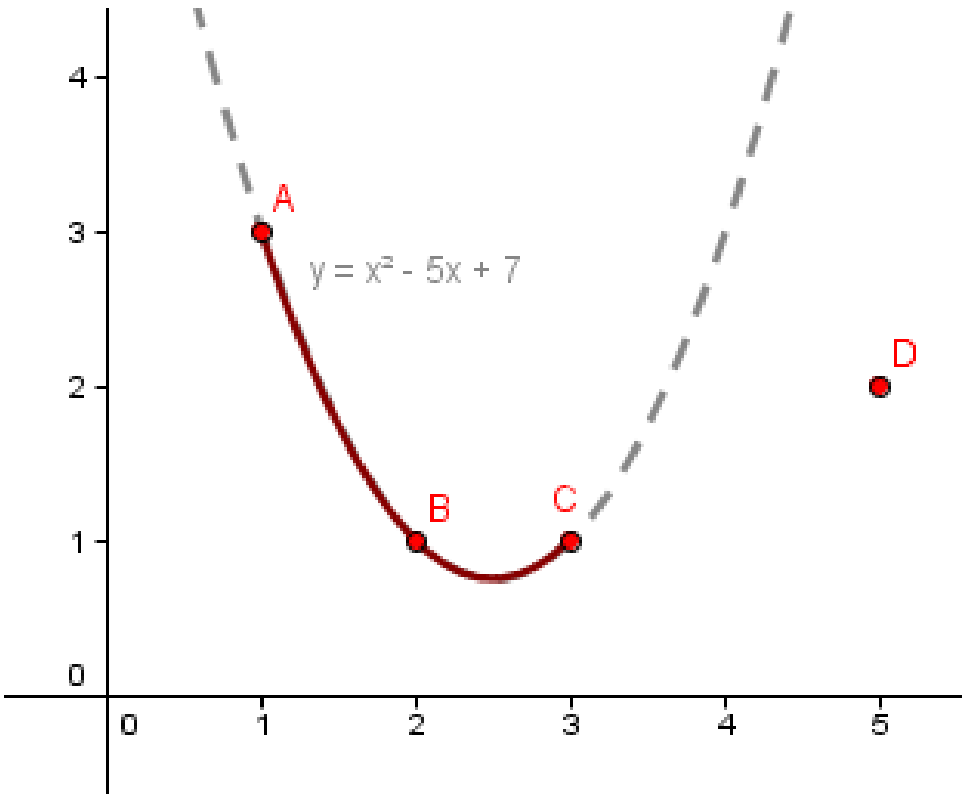
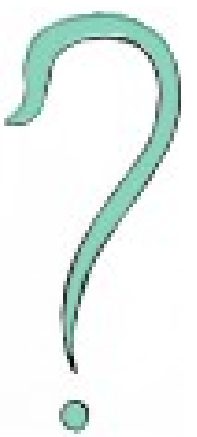
- We know: $f(1) = 3, \; f(2) = 1, \; f(3) = 1$
- **3 unknowns, 3 constraints, we can solve it.**
- http://www.wolframalpha.com/input/?i=a*1+%2B+b*1+%2B+c+%3D+3%2C+a*4+%2B+b*2+%2B+c+%3D+1%2C+a*9+%2B+b*3+%2B+c+%3D+1

Curves

- What choices we have with 4 points?





One additional point meant another line, could we have 2 parabolas here?

Curves

- Contstraints do not have to be on the function.

Curves

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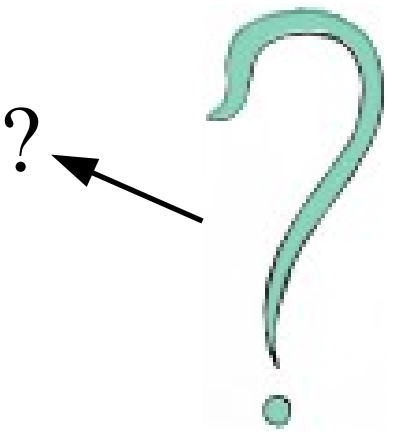
- They can also be on the derivative of it.

$$g(x)=a_2\cdot x^2+a_1\cdot x+a_0 \qquad g'(x)=?$$



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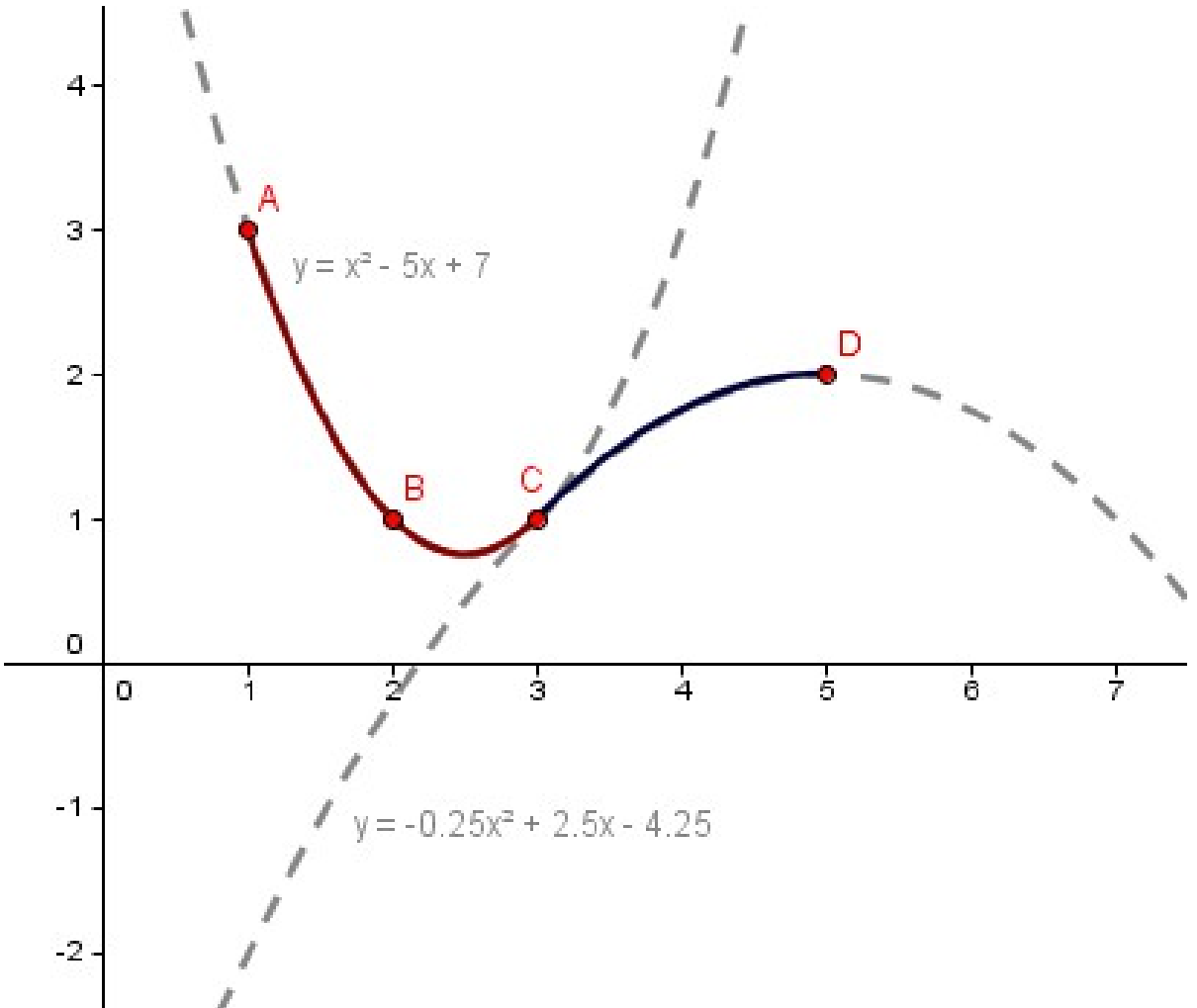
Curves

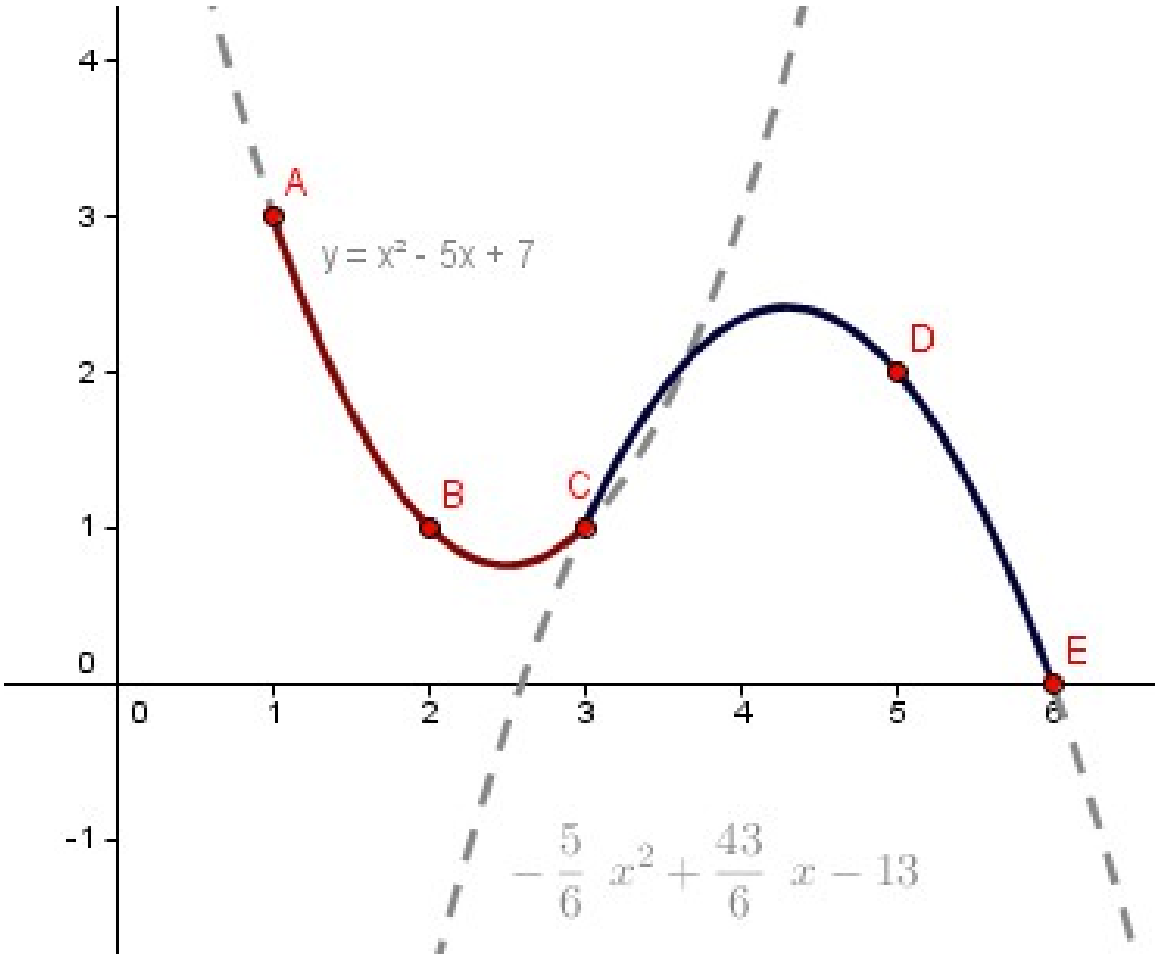


Smoothness

- What if we have 5 points and we put two parabolas through them without accounting for the derivative?

Smoothness

- That spline is not C^1 smooth.



Smoothness (continuity)

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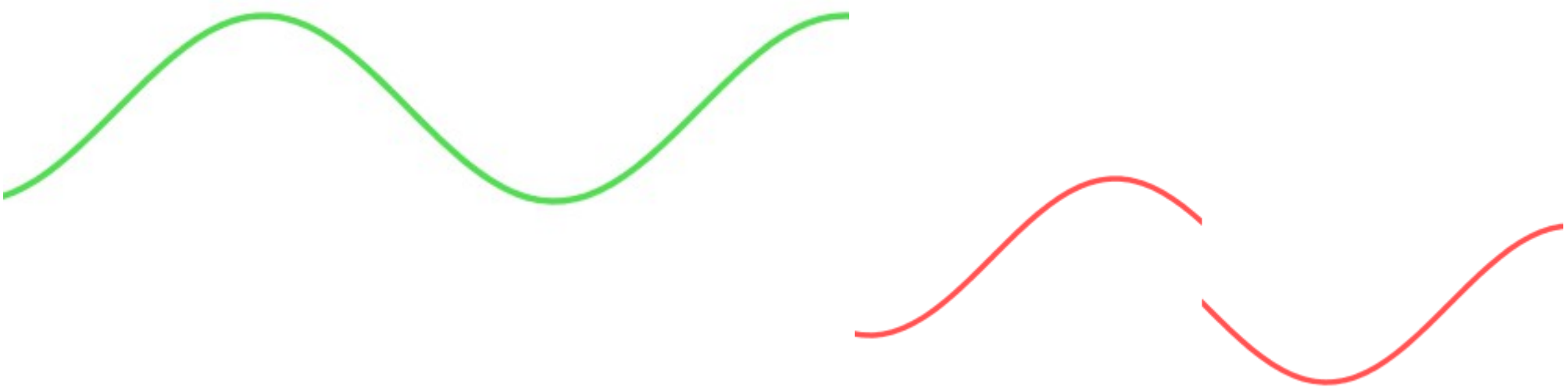
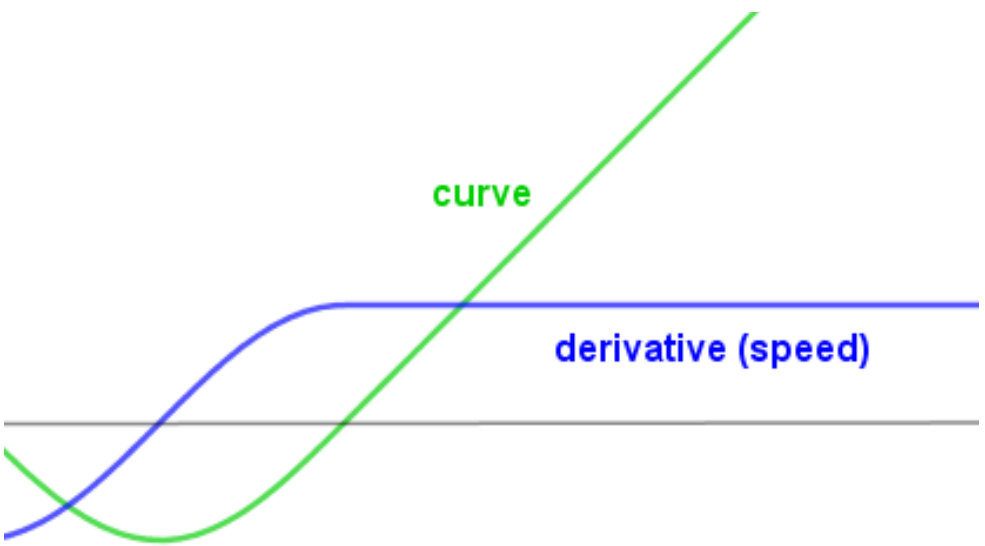
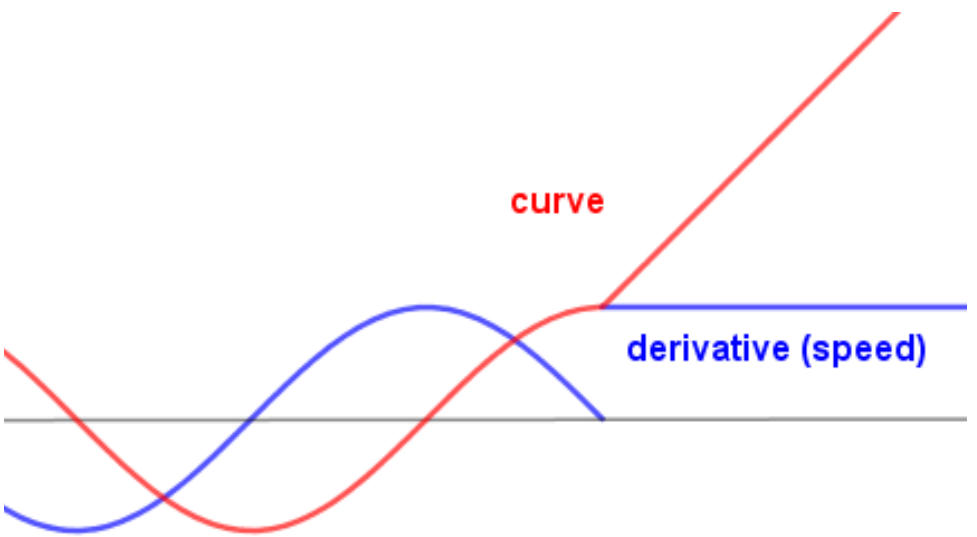
- Different levels of smoothness:
  - C^0 – Curve itself is continuous



Smoothness (continuity)

- Different levels of smoothness:
  - C^0 – Curve itself is continuous
  - C^1 – First derivative (speed) is continuous





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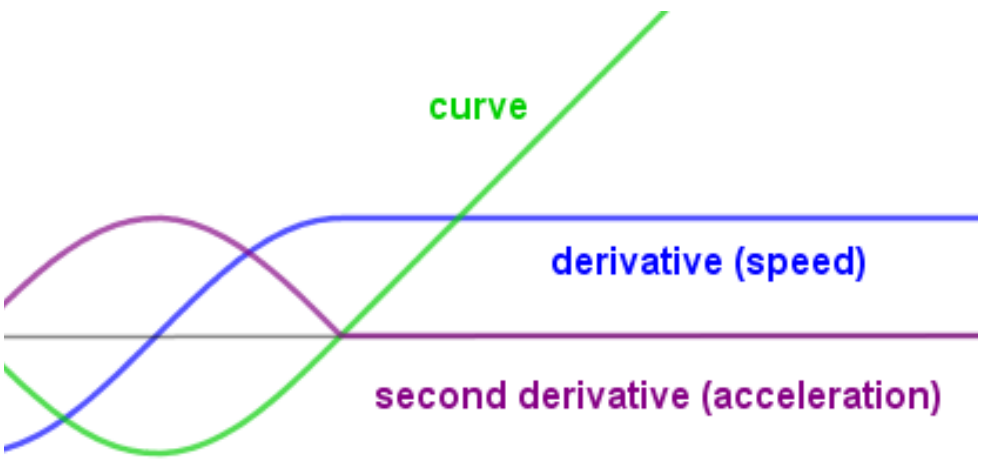

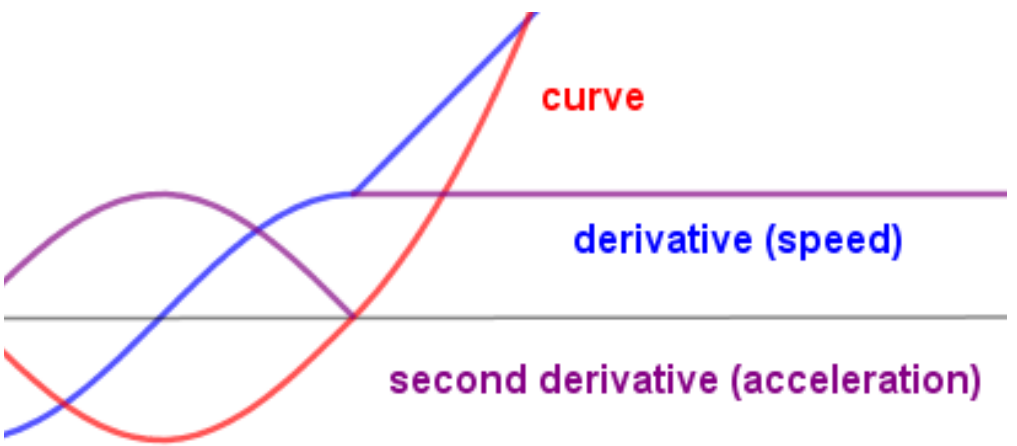

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- Often times C^1 or C^2 smooth curves are enough in computer graphics.
- If we put quadratic curves together, so the spline is C^1 smooth, how to get C^2 smoothness?

Find the second derivatives of our previous example...

Parametric Curves

- Implicit form: $f(x) = a_2 \cdot x^2 + a_1 \cdot x + a_0$
  - Good for testing points in a curve
  - Finding collisions

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For your regular mathematical quadratic fun.

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- Parametric form: $g(t)=(t+x_0,\ a_2 \cdot t^2+y_0)=(x,\ y)$

$$x_0=\frac{-a_1}{2 \cdot a_2},\quad y_0=f(x_0)$$

  - Good for **generating points on the curve**

Parametric form of a quadratic polynomial: http://www.nabla.hr/PC-ParametricEqu2.htm

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  - Good for generating points on the curve
  - **What other parametric equations you know?**

Parametric Curve Construction

- We want to find the **vector coefficients** a_i for a **function of *t* (time)**, where $t \in [0..1]$

Quadratic: $$curve(t) = a_0 + a_1 \cdot t + a_2 \cdot t^2$$

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- For a quadratic curve in 2D we need 3 points. Each 2D point gives 2 1D constraints.

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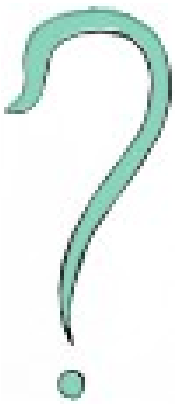

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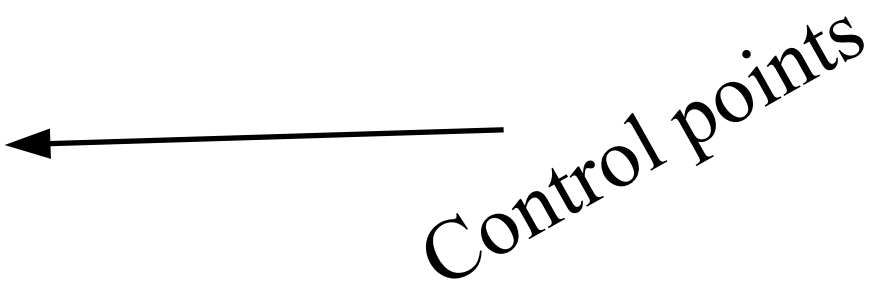
$$curve(1) = (3, 1) = \boldsymbol{p_2}$$



These values are based on our example from before.

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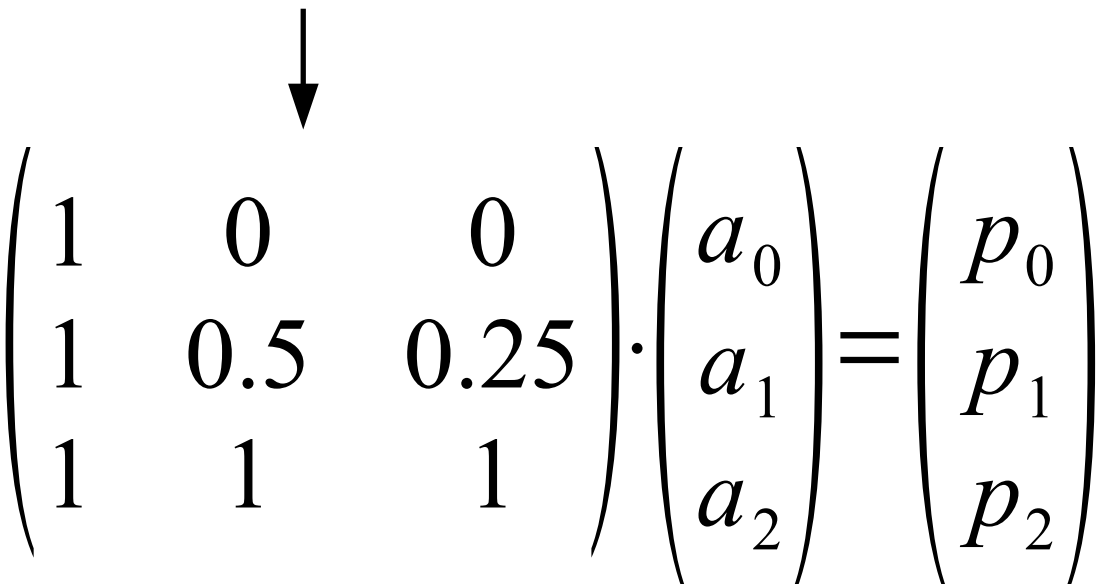
- Usually the system of constraints is written in a **constraint matrix**.

Parametric Curve Construction

- Our constraint matrix



Parametric Curve Construction

- Our constraint matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0.5 & 0.25 \\ 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \end{pmatrix}$$

- Write out the equations to see, that this is exactly what we did before with the implicit eq.

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Only that a_i and p_i are vectors too now.

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Parametric Curve Construction

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In short: $C \cdot a = p$

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How to find $\boldsymbol{a}$?

- Write out the equations to see, that this is exactly what we did before with the implicit eq.

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- We can find $a = C^{-1} p$, the inverse constraint matrix C^{-1} is often denoted B and called the **basis / blending matrix**.

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In our example:

$$B = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 4 & -1 \\ 2 & -4 & 2 \end{pmatrix}$$

Inverse (and other stuff) matrix calculator: http://www.bluebit.gr/matrix-calculator/

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Parametric Curve Construction

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Parametric Curve Construction

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$$curve(t) = b_{0,0}\,p_0 + b_{0,1}\,p_1 + b_{0,2}\,p_2 + \\ + b_{1,0}\,p_0\,t + b_{1,1}\,p_1\,t + b_{1,2}\,p_2\,t + \\ + b_{2,0}\,p_0\,t^2 + b_{2,1}\,p_1\,t^2 + b_{2,2}\,p_2\,t^2$$

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Coefficients from one (*i*-th) column of the matrix B.

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The functions $\boldsymbol{b_i}$ are called **basis / blending functions**.

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1) Pick a degree of the curve
2) Fix the parameters (*incl* control points)
3) Create the constraint matrix C
4) Find the basis matrix $B = C^{-1}$
5) Read the blending functions from the basis matrix

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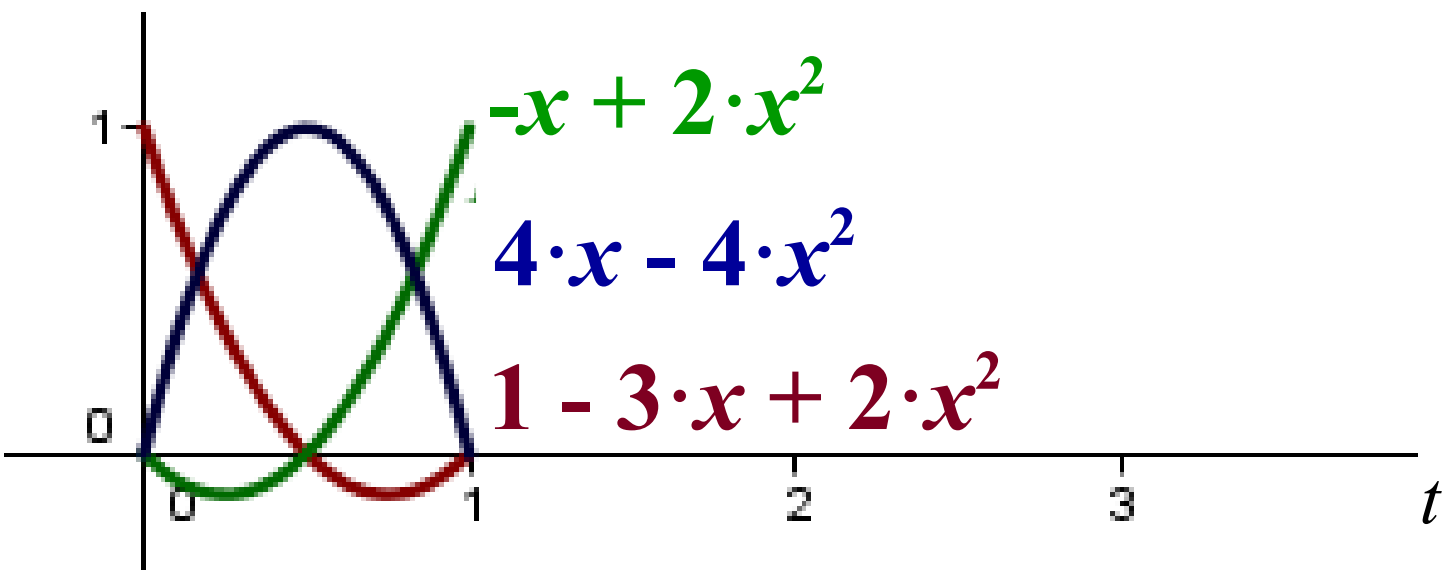
- Here are the found blending functions for interpolating a quadratic curve between 3 points:



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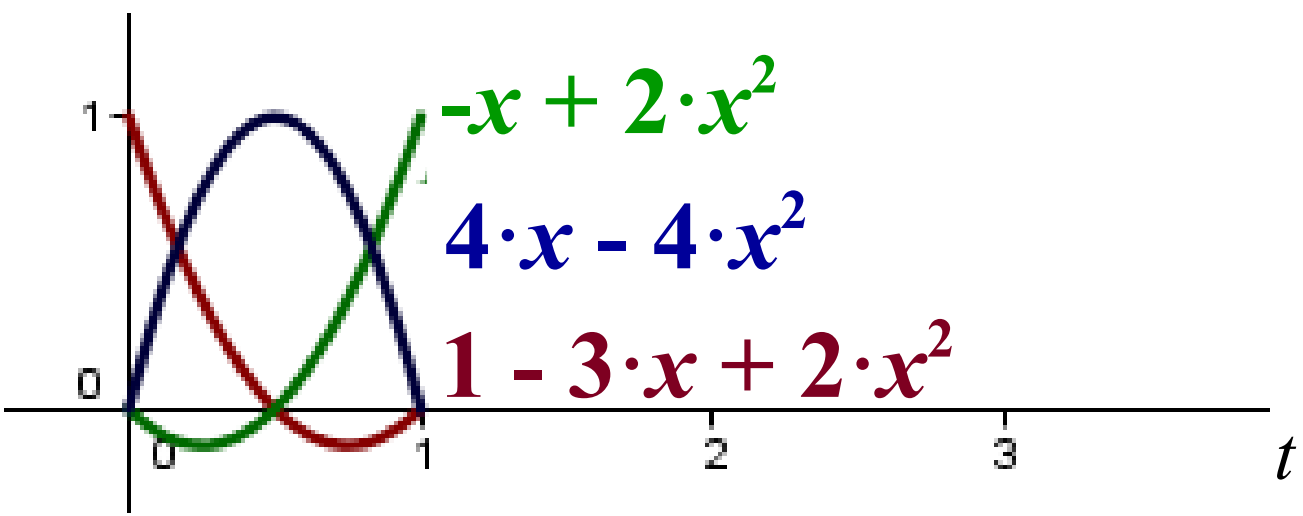

- Different constraints give different functions

Cubic not Quadratic

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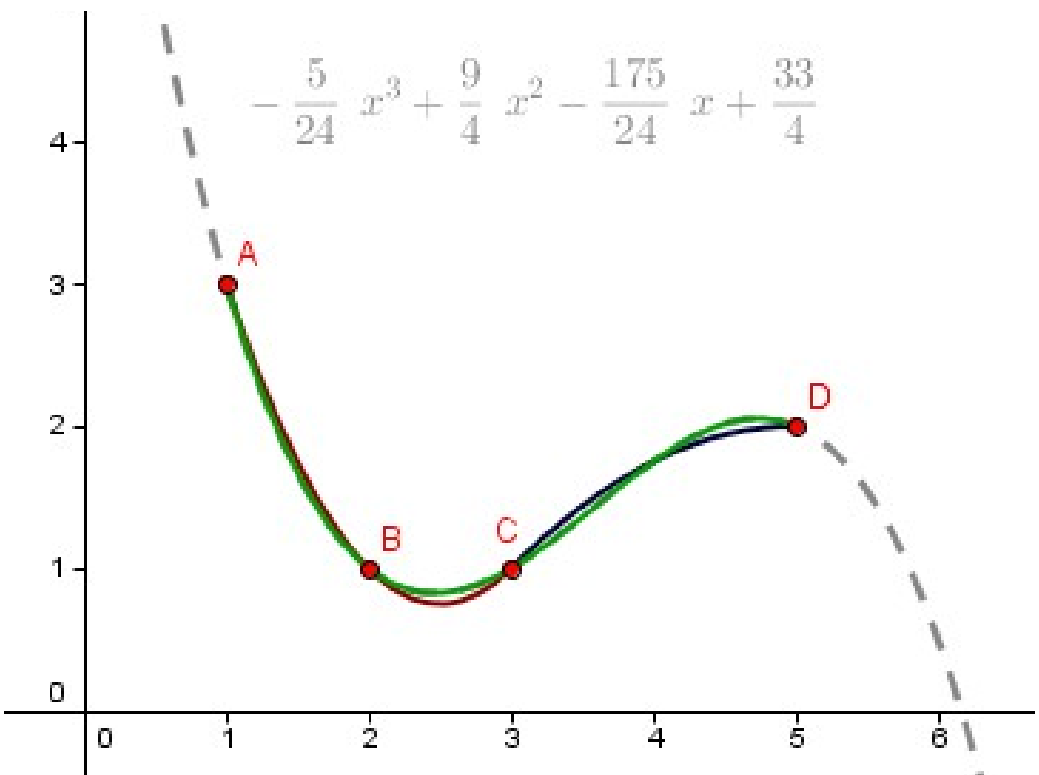
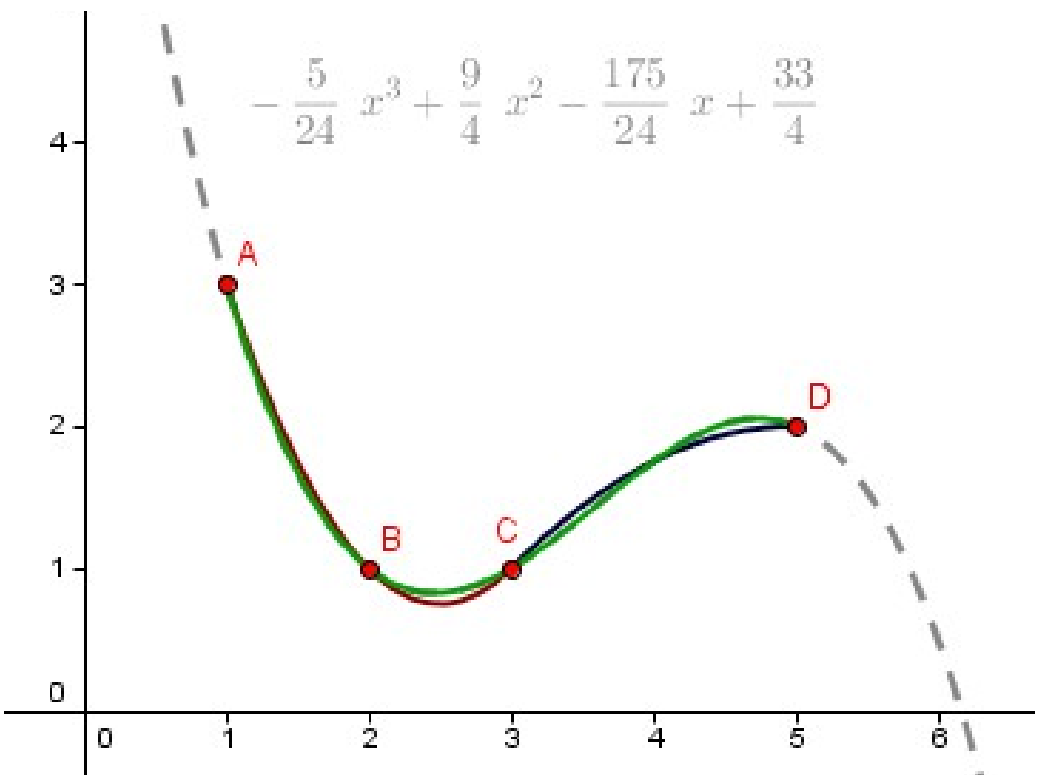


Cubic not Quadratic

- In computer graphics, we usually want to use cubic polynomials, not quadratics.
- Cubic polynomials provide 4 possible constraints.
- Splines can achieve C^2 smoothness.



Hermite Spline

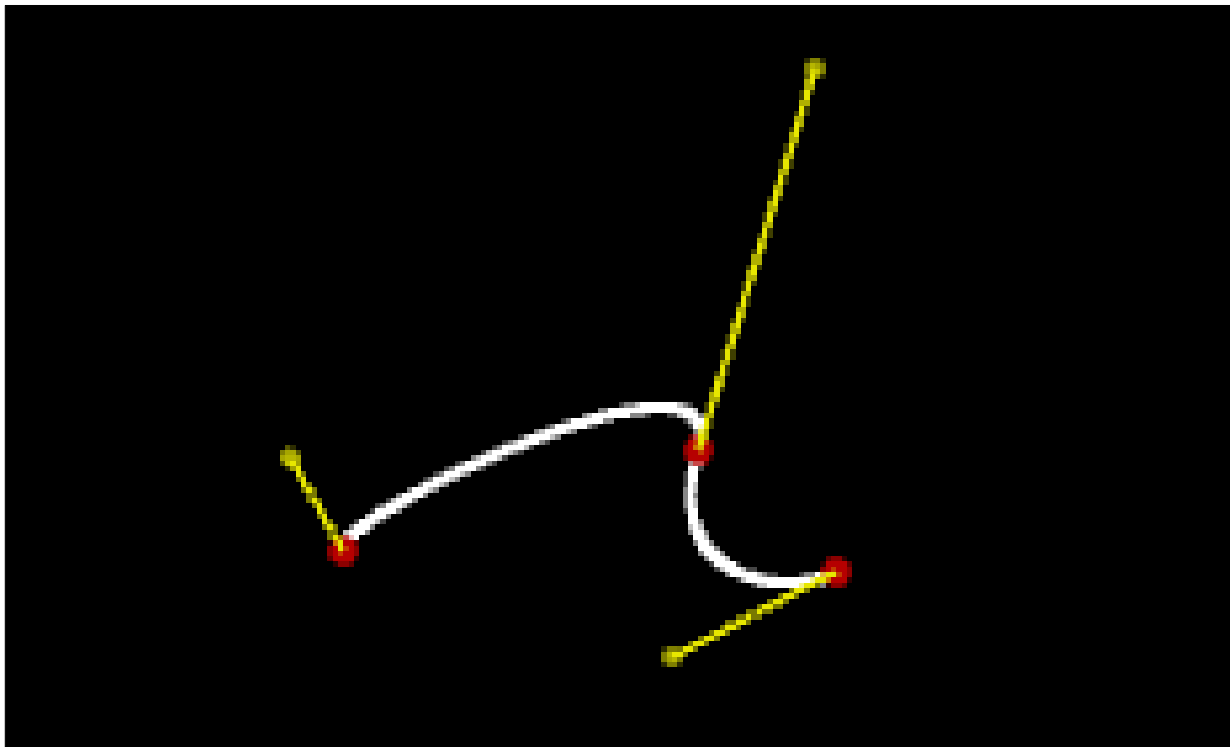
- The derivatives at the endpoints are parameters.
- Segments share the endpoints and derivatives.

$$curve(0) = p_0$$

$$curve'(0) = p_1$$

$$curve(1) = p_2$$

$$curve'(1) = p_3$$



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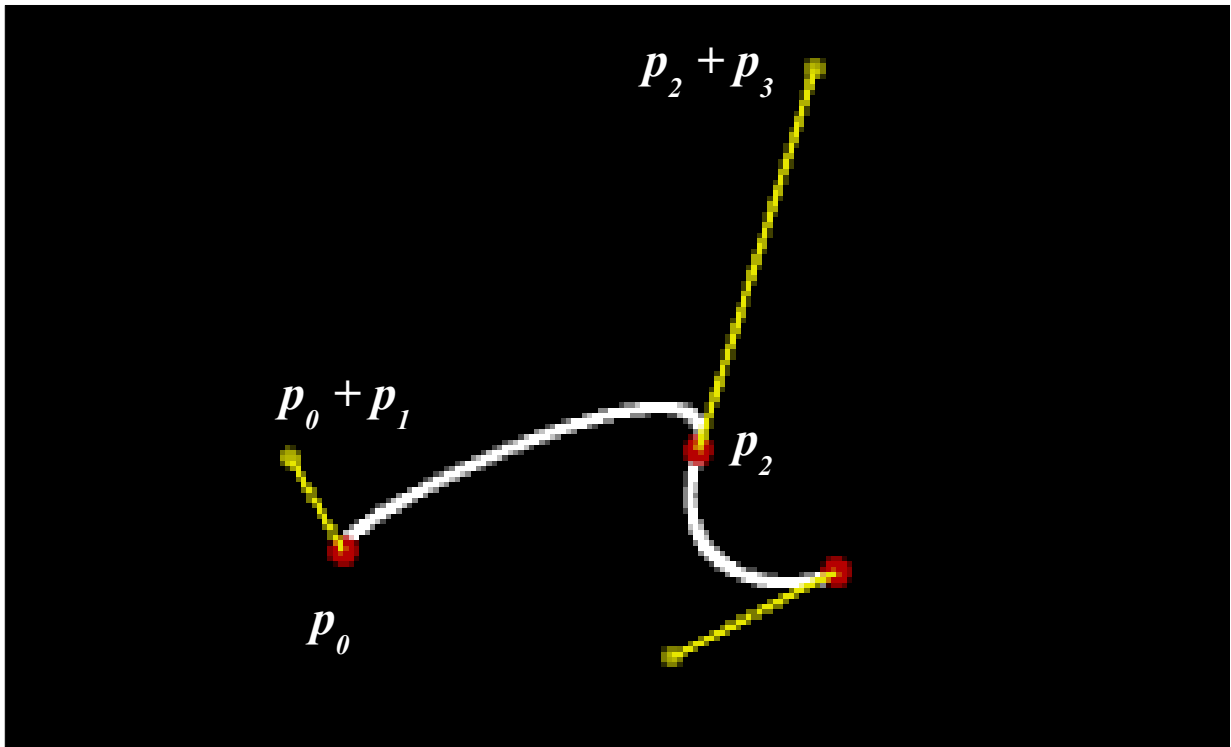

Catmull-Rom Spline

- We interpolate the p_1 and p_2.
- Dervatives are calculated using the other points.

$$curve'(0) = 0.5 \cdot (p_2 - p_0)$$

$$curve(0) = p_1$$

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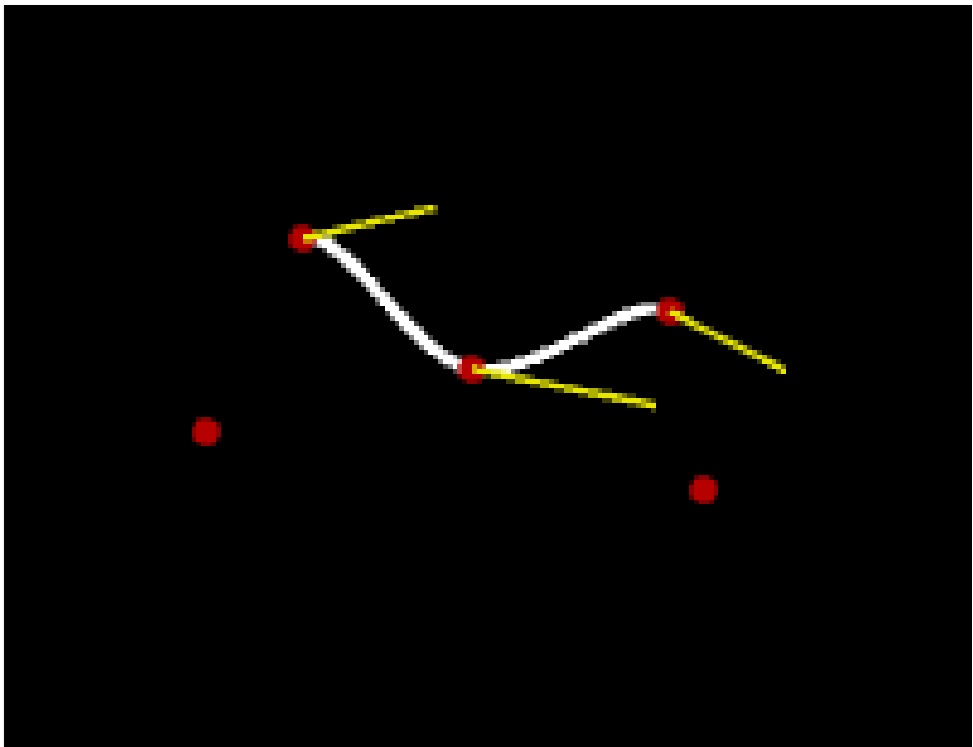
$$curve'(1) = 0.5 \cdot (p_3 - p_1)$$



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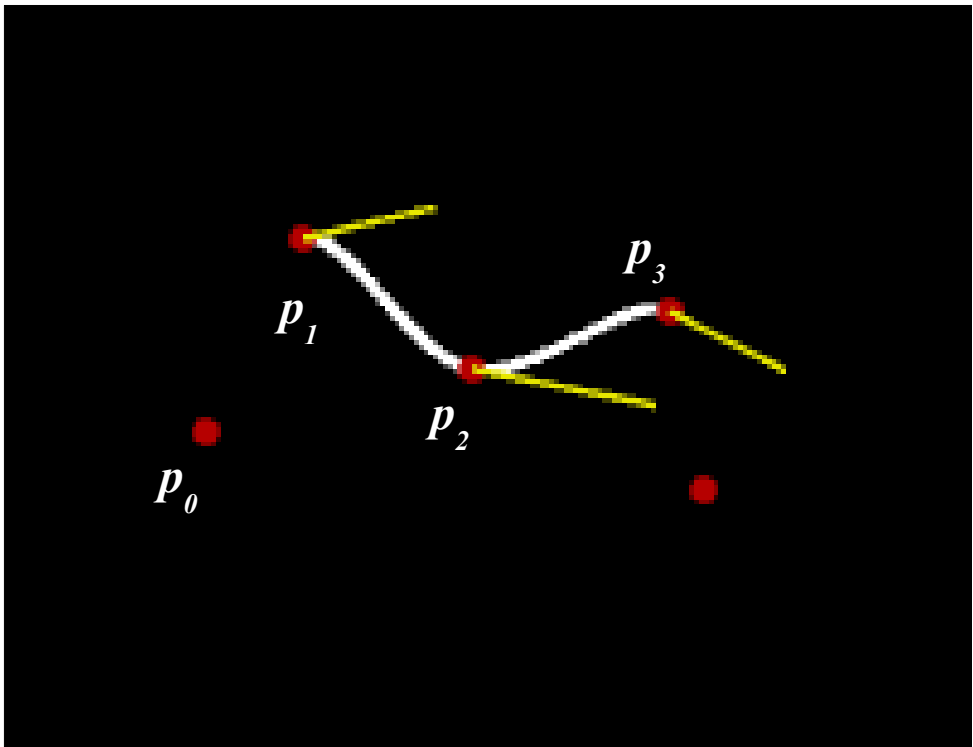
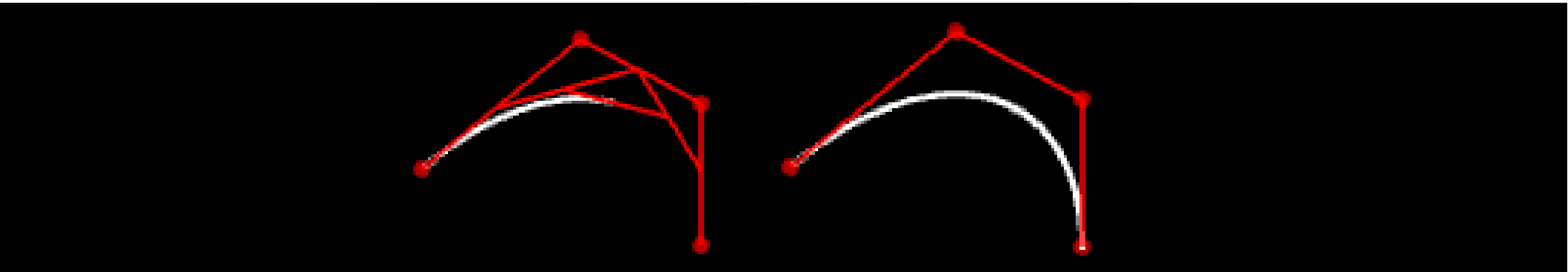


Only need to specify spline's start and end derivatives

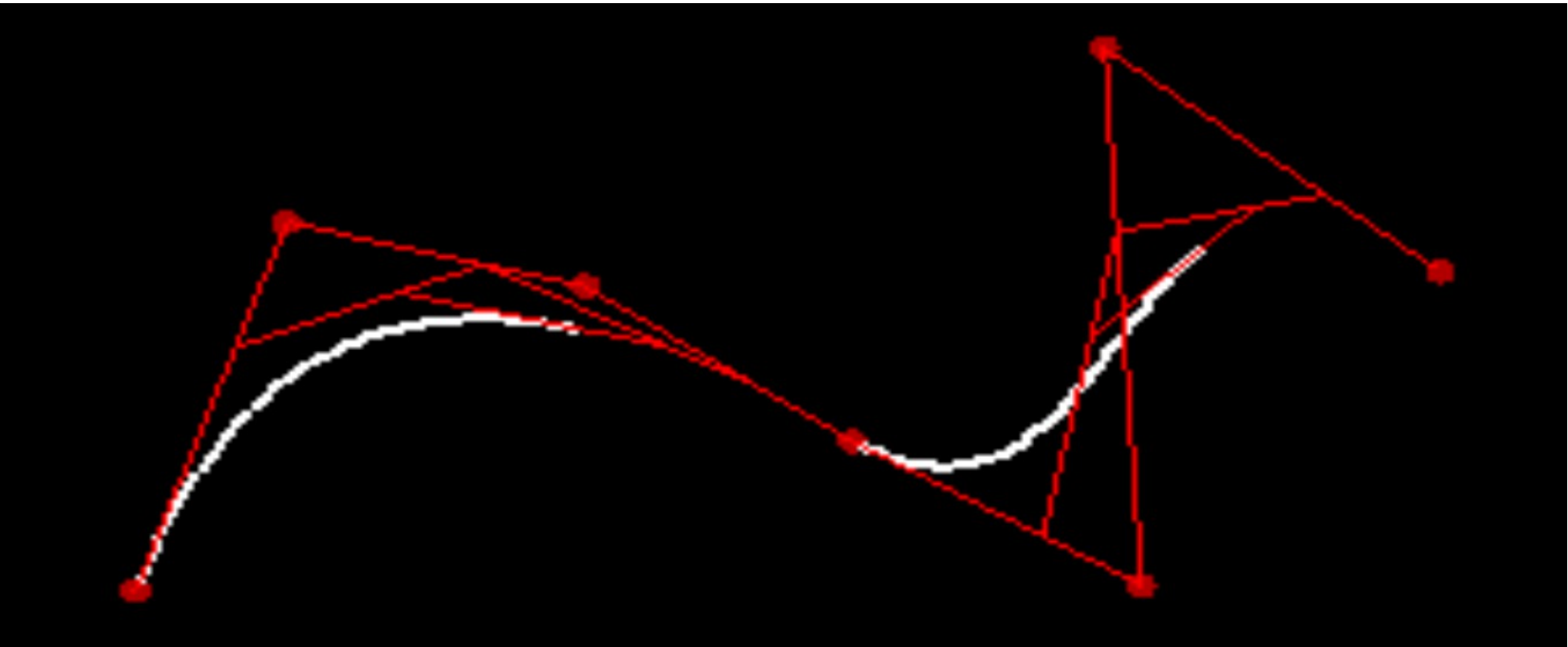
Bezier Curve

- Can be constructed using the constraints and finding the blending functions.
- Can also be constructed in a procedural way:
  - Subdivide the lines connecting the control points, into proportions t and $(1-t)$.
  - Recurse until you get a point on the curve.



Bezier Curve

- That procedure is called De Casteljau's algorithm.



Bezier Curve

- That procedure is called De Casteljau's algorithm.
- The corresponding blending functions are called Bernstein basis polynomials.

$$b_{0,0}(t)=1$$

$$b_{0,1}(t)=1-t,\quad b_{1,1}(t)=t$$

$$b_{0,2}(t)=(1-t)^2,\quad b_{1,2}(t)=2\cdot t\cdot(1-t),\quad b_{2,2}(t)=t^2$$

$$b_{i,\,degree}(t)=\binom{degree}{i}\cdot t^i\cdot(1-t)^{degree-i}$$

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Already familiar?

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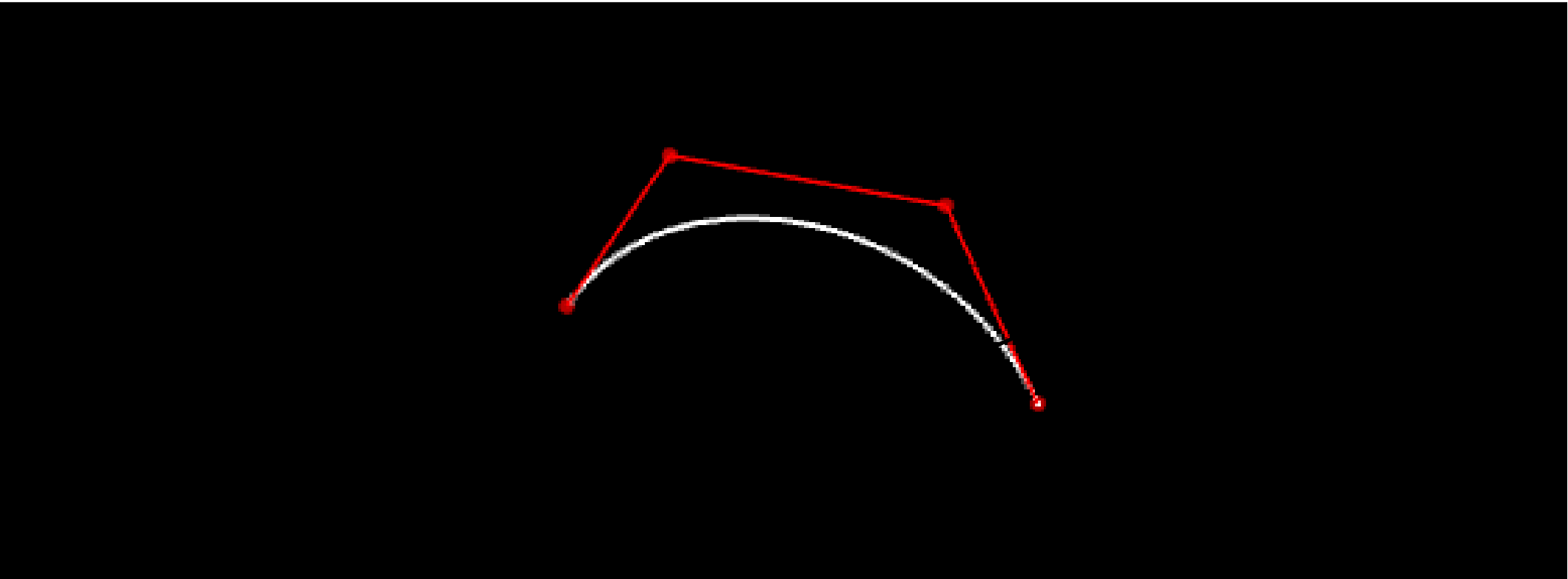
Those you used in the Imported Chopper task.

Bezier Curve

- Always inside the convex hull of the control points.



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- Always inside the convex hull of the control points.
- Affine invariance – affine transformations on the control points, transform the curve itself correctly too.
- Sufficiently smooth splines can be constructed (Stärk's construction, we will see in the B2 practice)
- Very widely used (*eg* font rendering)

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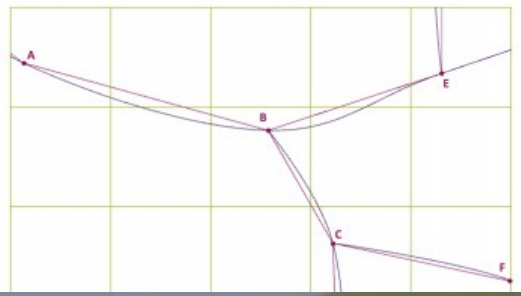
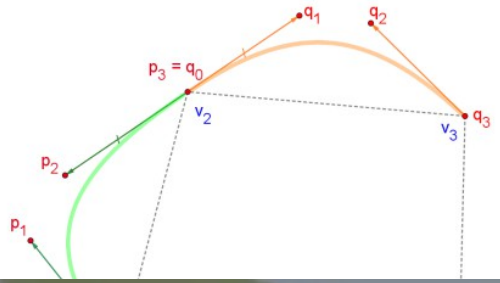
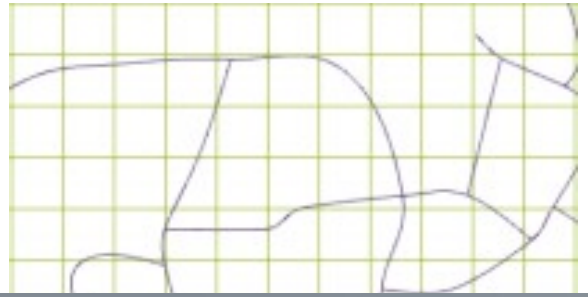
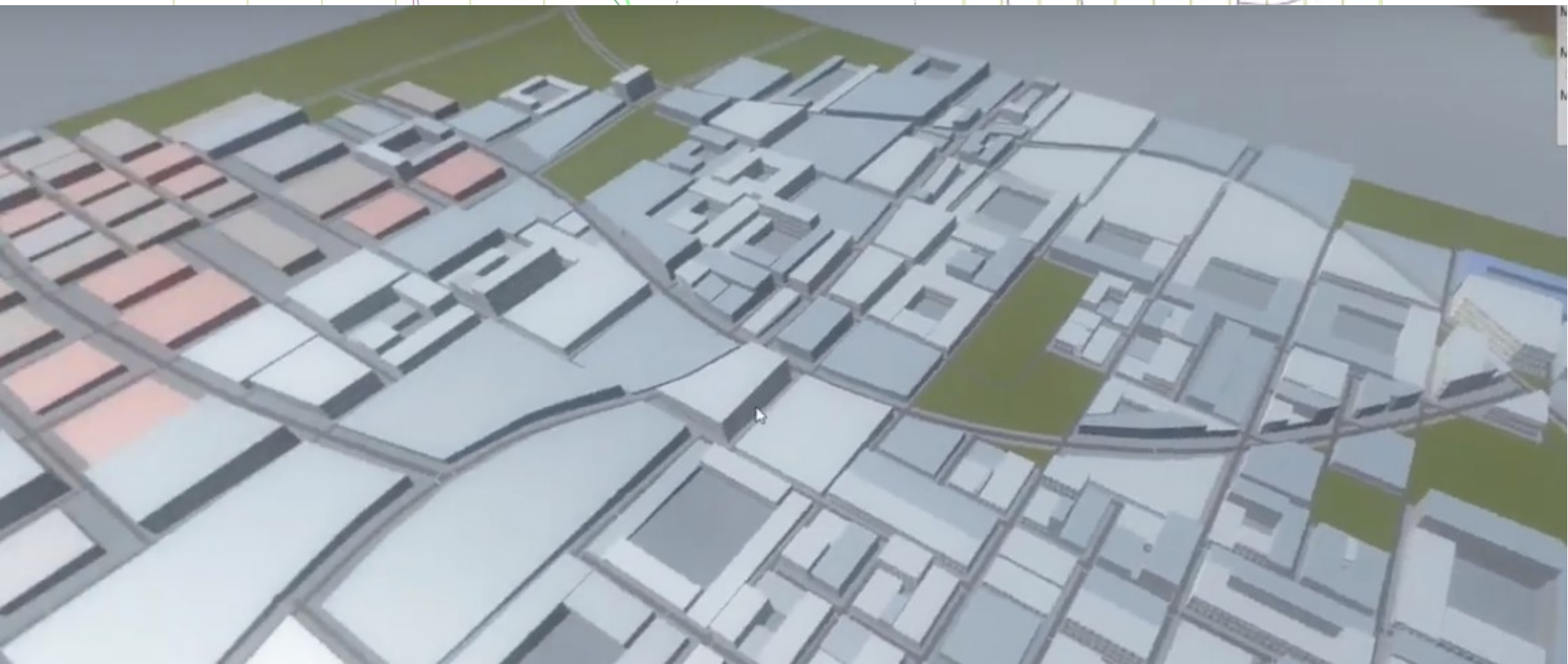
- Hermite and Catmull-Rom – are not C^2 smooth.
- Bezier – does not interpolate the control points.

Infinite Procedural Infrastuctured World Generation by Andreas Sepp









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