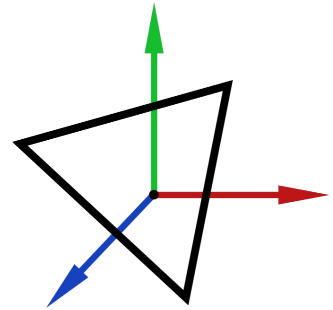
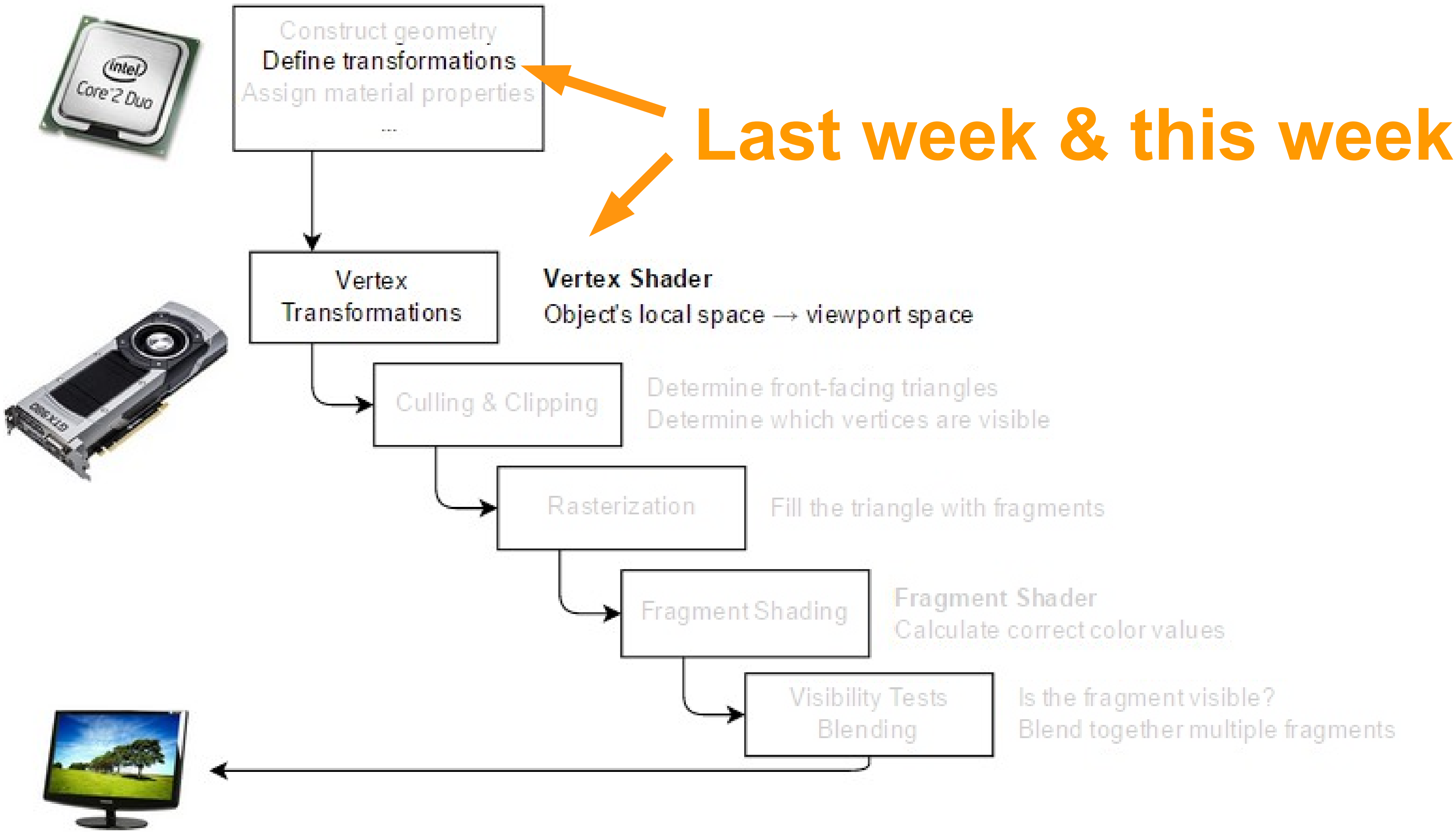


# Computer Graphics

## MTAT.03.015

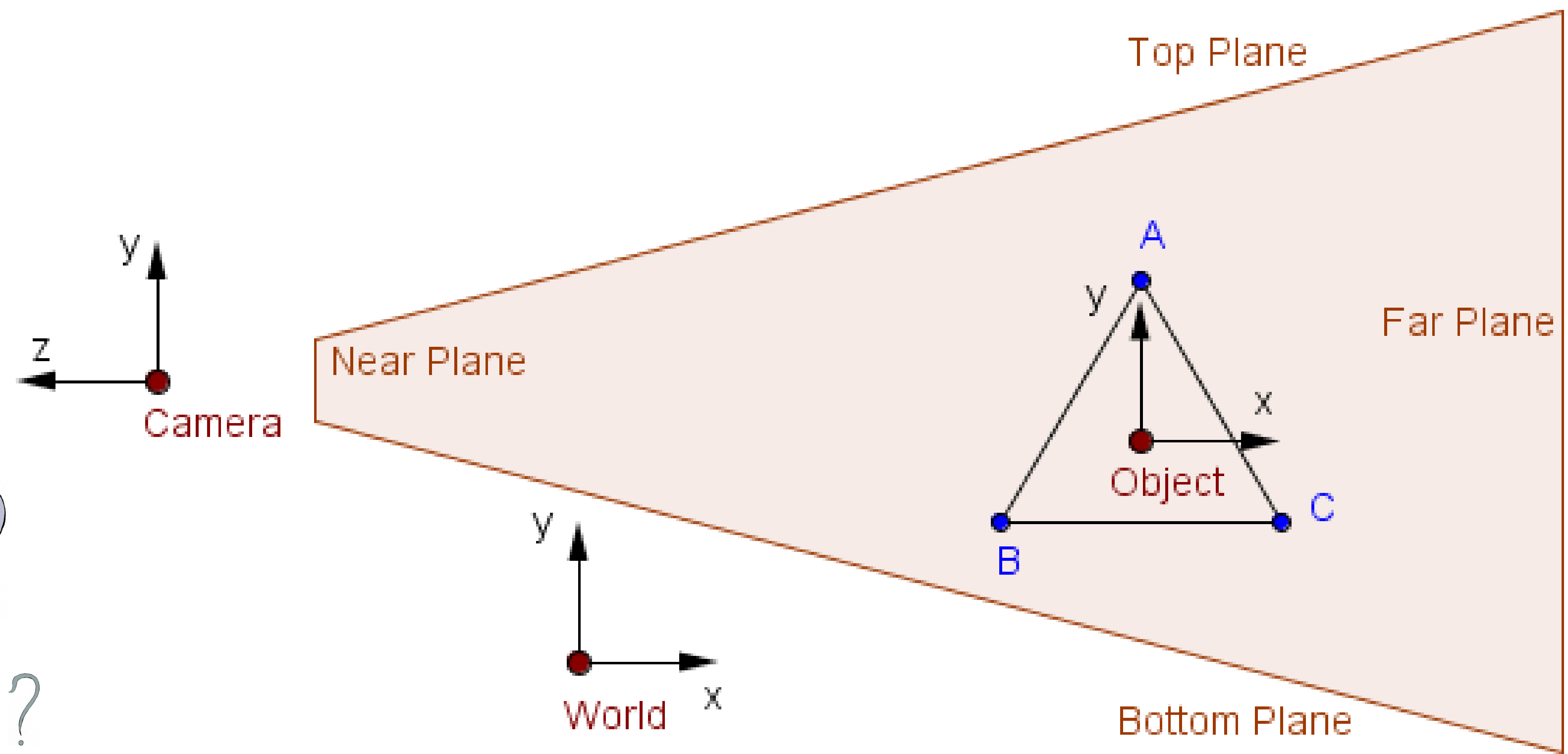
Raimond Tunnel

UNIVERSITY OF TARTU
Institute of Computer Science


Construct geometry
Define transformations
Assign material properties
...
Last week & this week
Vertex Transformations
Vertex Shader
Object's local space → viewport space
Culling & Clipping
Determine front-facing triangles
Determine which vertices are visible
Rasterization
Fill the triangle with fragments
Fragment Shading
Fragment Shader
Calculate correct color values
Visibility Tests
Blending
Is the fragment visible?
Blend together multiple fragments

# Frames of Reference

- Can you name different spaces (frames of reference) we use?

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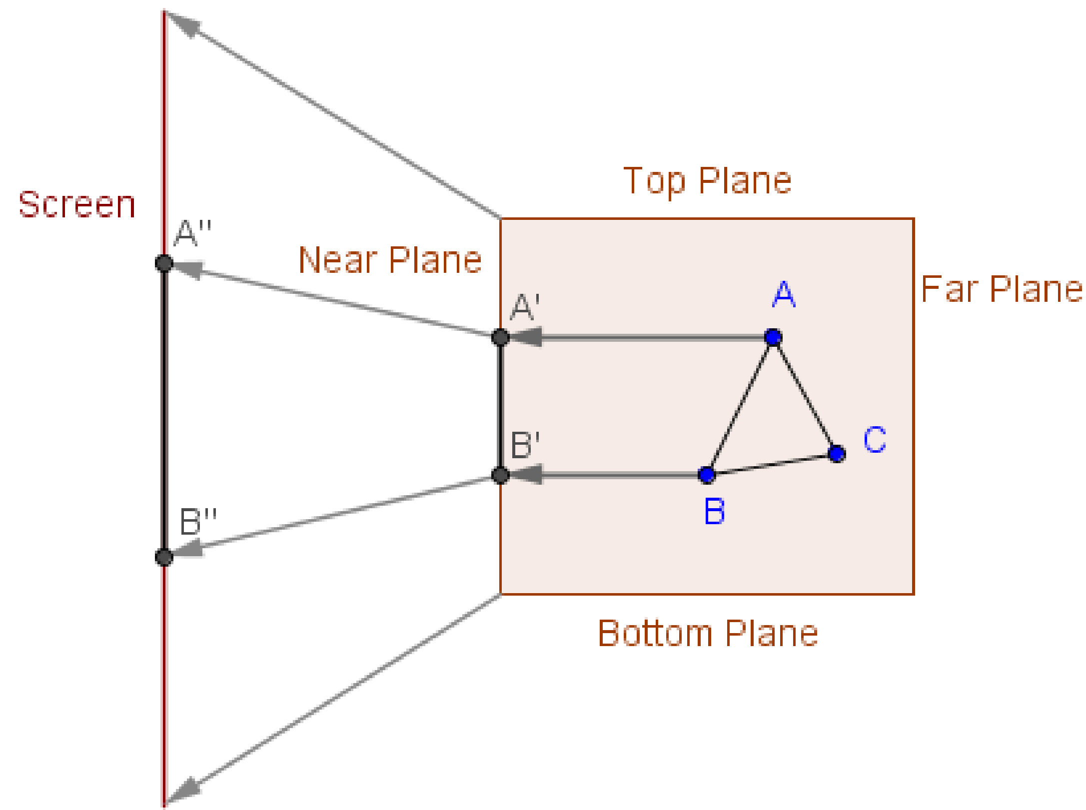

# Object Space → World Space

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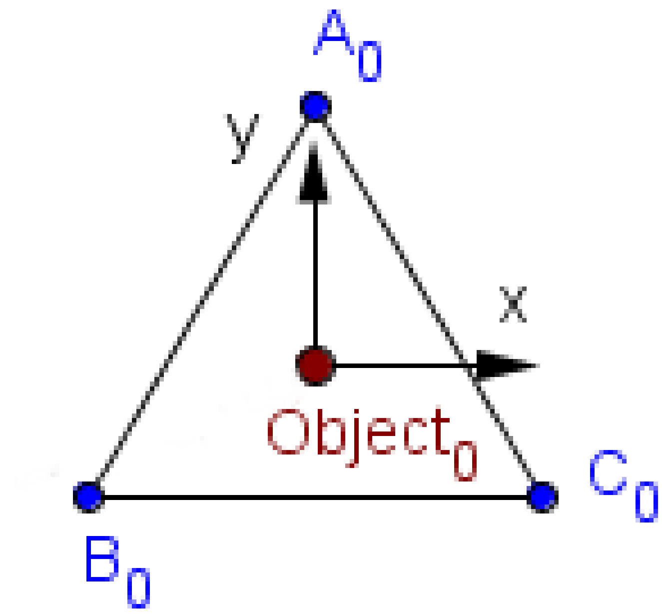
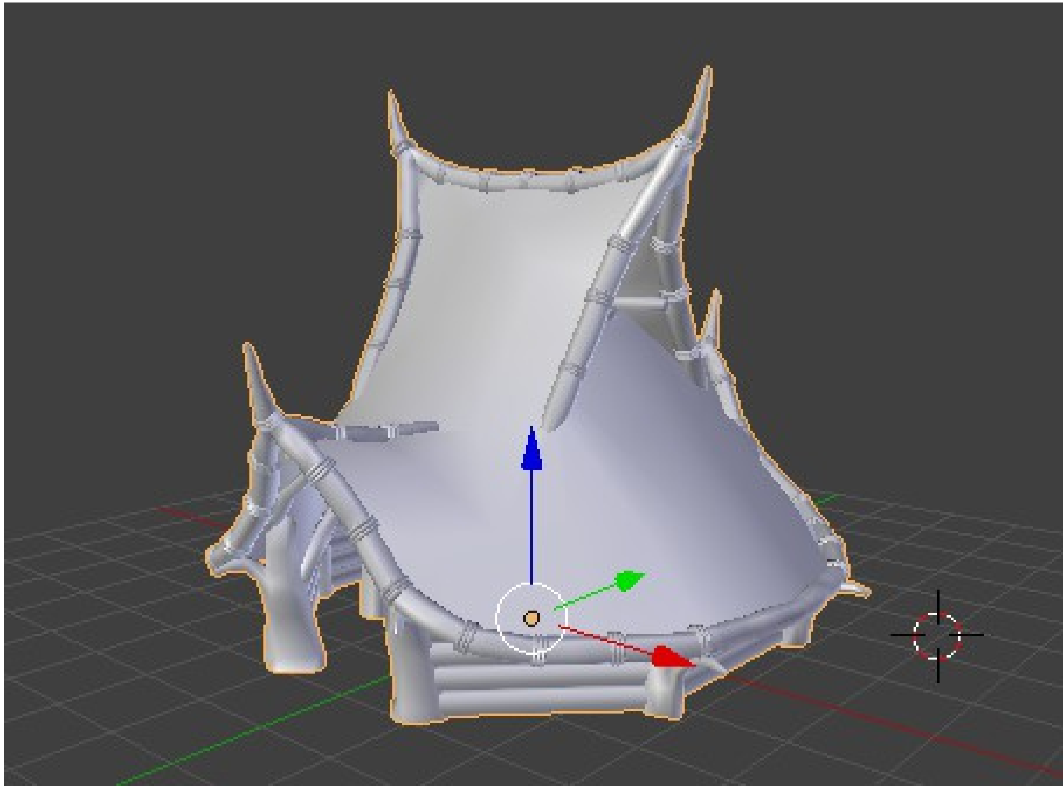
  - *Symmetrically* from the origin



# Object Space → World Space

- We model our objects in object space
    - *Symmetrically* from the origin
    - Up from the origin



# Object Space → World Space

- We model our objects in object space
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- We position, orient and scale our object with the **model matrix**, thus creating the world space!

$$\text{projectionMatrix} \cdot \text{viewMatrix} \cdot \text{modelMatrix} \cdot v$$

$$P \cdot V \cdot M \cdot v$$

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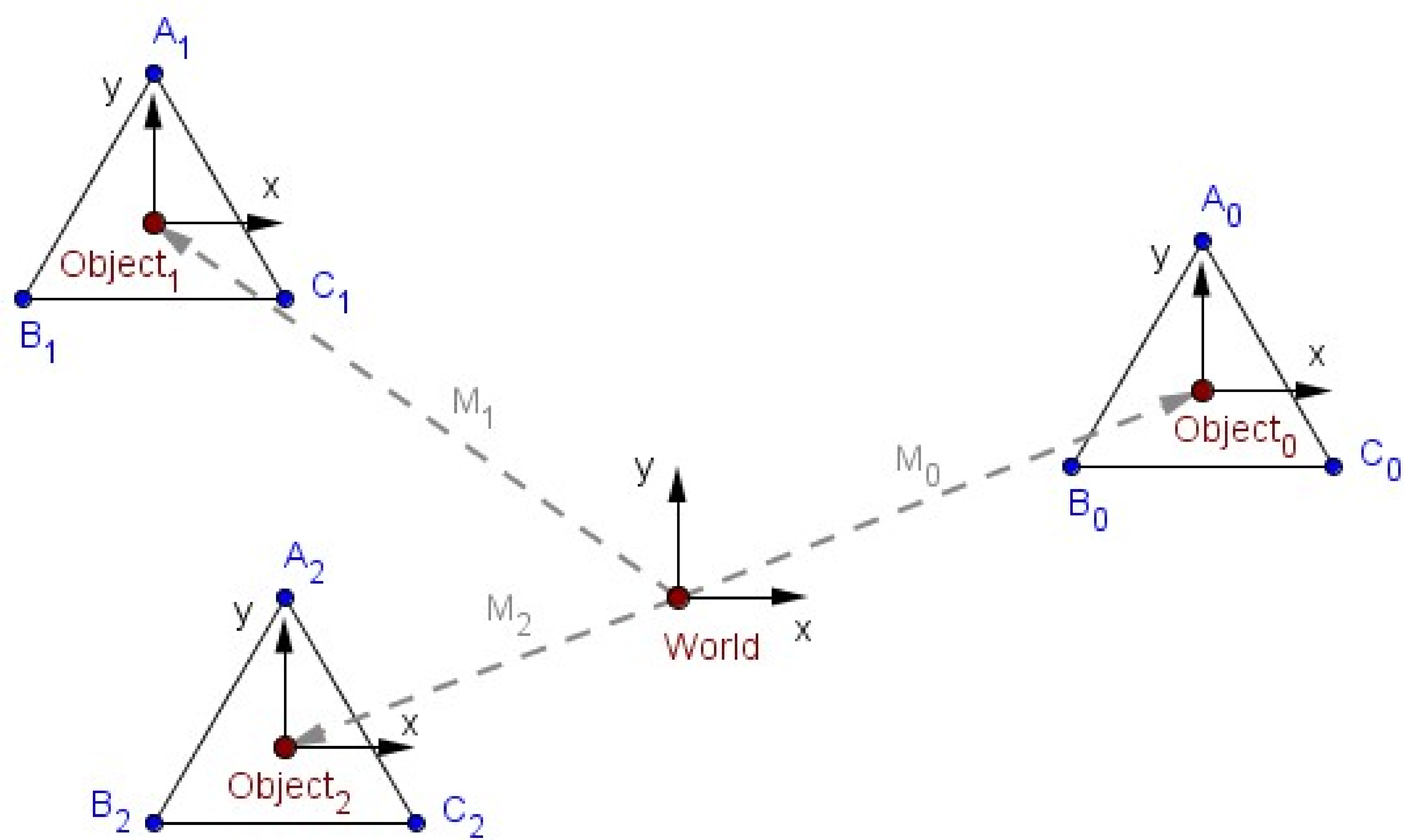
  - Origin defined by the identity transformation
  - Every child transformed relative to it

# Object Space → World Space




This is what you did last week. :)

# Object Space → World Space

$$\text{projectionMatrix} \cdot \text{viewMatrix} \cdot \text{modelMatrix} \cdot v$$

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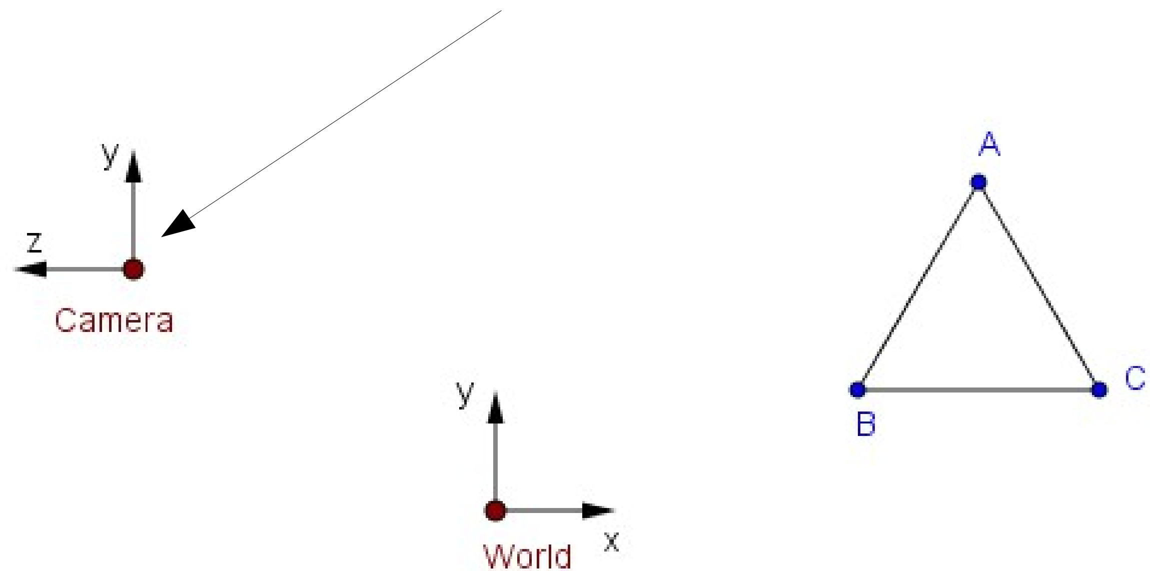
This is what you did last week. :)

# World Space → Camera Space

- We want to represent everything related to the camera (to make projection easier)

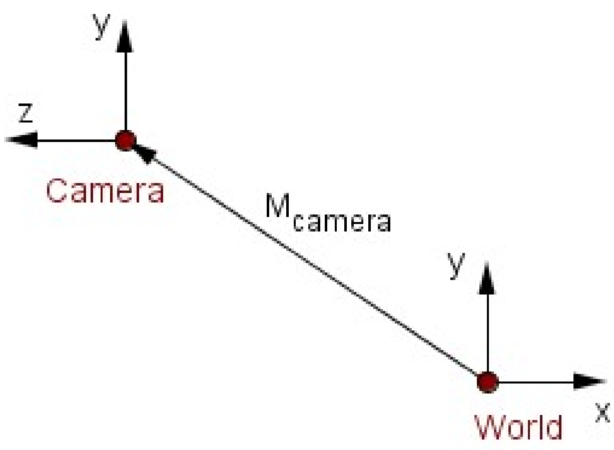
Transform so that this is the origin + basis

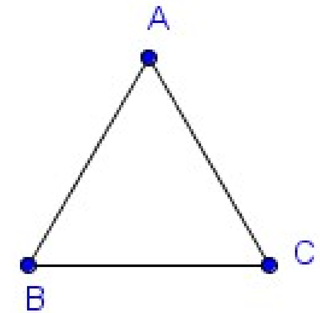



Origin & basis – *reeper* (est)

# World Space → Camera Space

- We want to represent everything related to the camera (to make projection easier)
- Think of the camera as another object in the scene.



# World Space → Camera Space

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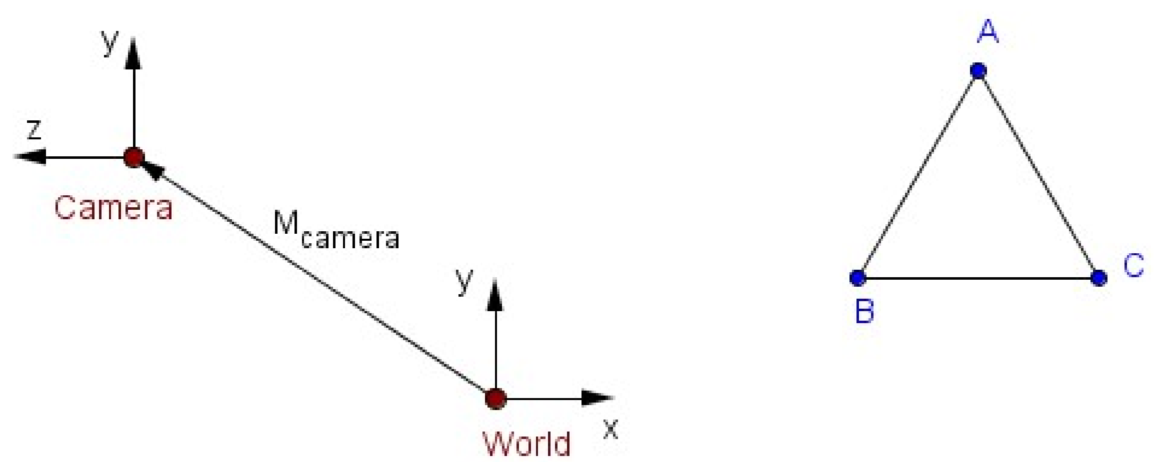
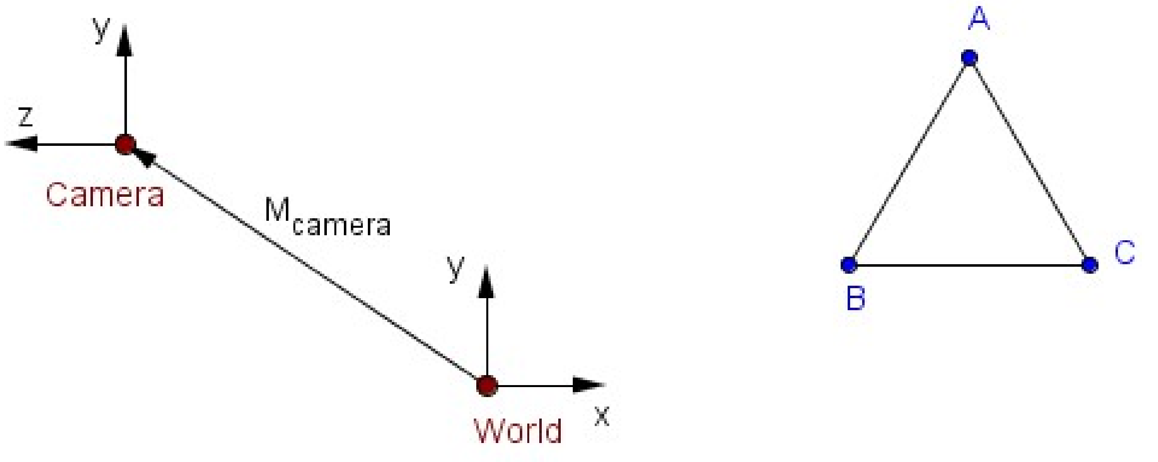
  - It has its own **rotation** and **position**.



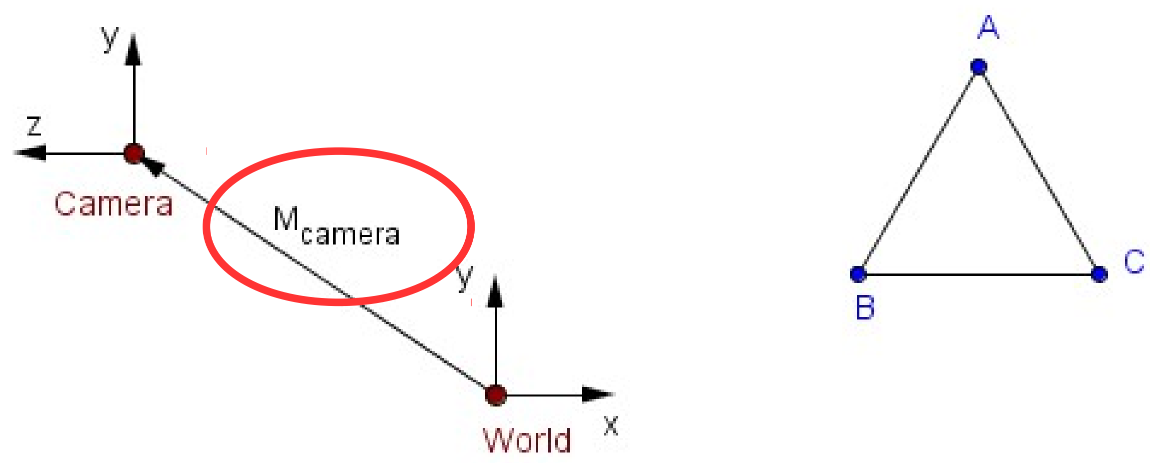
# World Space → Camera Space

- We want to represent everything related to the camera (to make projection easier)
- Think of the camera as another object in the scene.
  - It has its own **rotation** and **position**.
  - Scale is not relevant for the camera.



# World Space → Camera Space

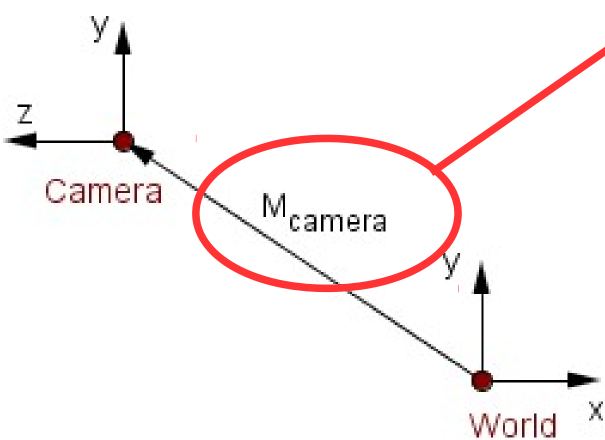
- Assume that we have a camera's model transformation matrix:



# World Space → Camera Space

- Assume that we have a camera's model transformation matrix:

$$M_{camera} = \begin{pmatrix} right_x & up_x & back_x & pos_x \\ right_y & up_y & back_y & pos_y \\ right_z & up_z & back_z & pos_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



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- Remember from last week that the columns are the transformed standard basis...

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- Remember from last week that the columns are the transformed standard basis...
- Can you come up with a matrix to transform our world relative to the camera?

# World Space → Camera Space

- **View matrix** can be found like this:

# World Space → Camera Space

- **View matrix** can be found like this:
  0) Camera's linear transform is an orthonormal matrix

$$M_{camera} = \begin{pmatrix} right_x & up_x & back_x & pos_x \\ right_y & up_y & back_y & pos_y \\ right_z & up_z & back_z & pos_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# World Space → Camera Space

- **View matrix** can be found like this:
  0) Camera's linear transform is an orthonormal matrix
  1) Transpose it to find the inverse

$$\begin{pmatrix} right_x & up_x & back_x \\ right_y & up_y & back_y \\ right_z & up_z & back_z \end{pmatrix}^T = \begin{pmatrix} right_x & right_y & right_z \\ up_x & up_y & up_z \\ back_x & back_y & back_z \end{pmatrix}$$

This was hinted at in the Preliminary Math Tasks 12-14.

# World Space → Camera Space

- **View matrix** can be found like this:
  0) Camera's linear transform is an orthonormal matrix
  1) Transpose it to find the inverse
  2) Camera's translation can be inverted by negation

$$\begin{pmatrix} right_x & right_y & right_z \\ up_x & up_y & up_z \\ back_x & back_y & back_z \end{pmatrix}$$

$$-\begin{pmatrix} pos_x \\ pos_y \\ pos_z \end{pmatrix} = \begin{pmatrix} -pos_x \\ -pos_y \\ -pos_z \end{pmatrix}$$

# World Space → Camera Space

- **View matrix** can be found like this:
  3) Put the two inverse transformations together in the opposite order

$$V = \begin{pmatrix} right_x & right_y & right_z & 0 \\ up_x & up_y & up_z & 0 \\ back_z & back_y & back_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & -pos_x \\ 0 & 1 & 0 & -pos_y \\ 0 & 0 & 1 & -pos_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# World Space → Camera Space

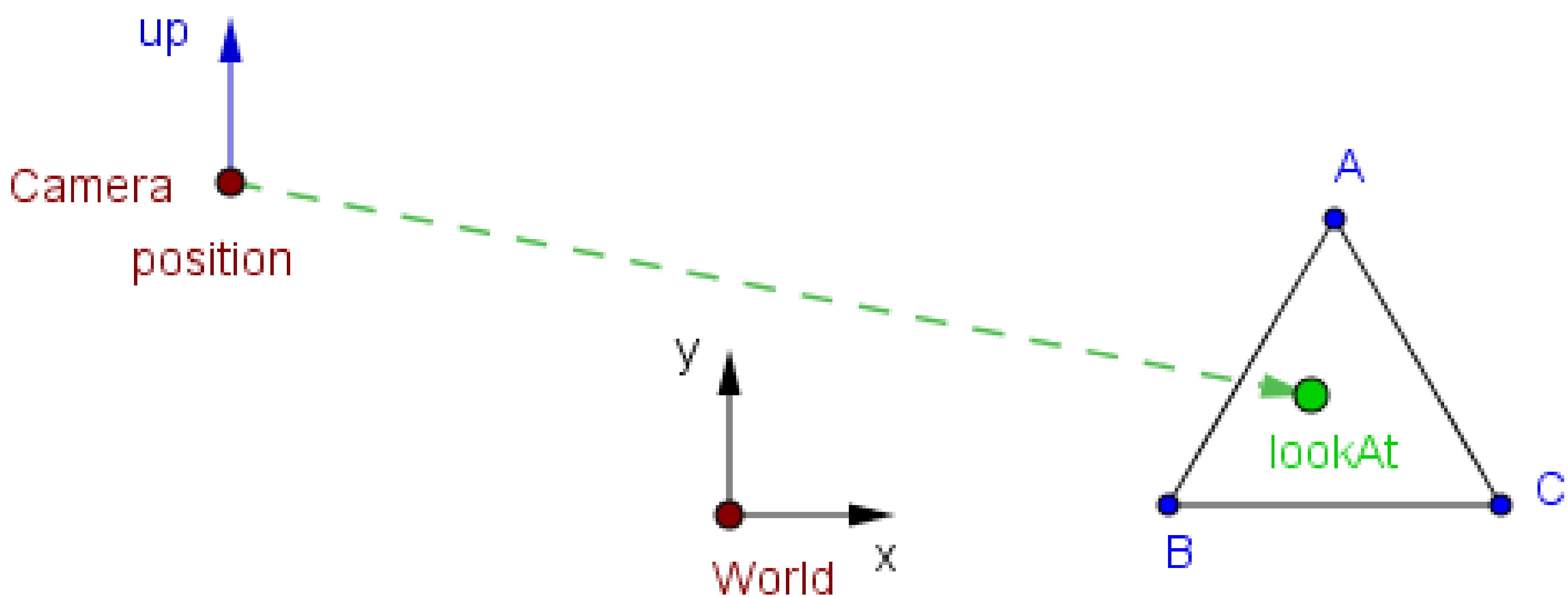
- **View matrix** can be found like this:

$$V = \begin{pmatrix} right_x & right_y & right_z & 0 \\ up_x & up_y & up_z & 0 \\ back_z & back_y & back_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & -pos_x \\ 0 & 1 & 0 & -pos_y \\ 0 & 0 & 1 & -pos_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1) Transpose the rotation to inverse it
2) Negate the translation to inverse it
3) Multiply together in the reverse order

# World Space → Camera Space

- Usually it is more intuitive to specify the camera by its **position**; **point it is looking at**; and the **up-vector**



# World Space → Camera Space

- Usually it is more intuitive to specify the camera by its **position**; **point** it is **looking at**; and the **up-vector**

**Three.js:**

```
camera.position.set(x, y, z);
camera.up.set(upX, upY, upZ);
camera.lookAt(point);
```

# World Space → Camera Space

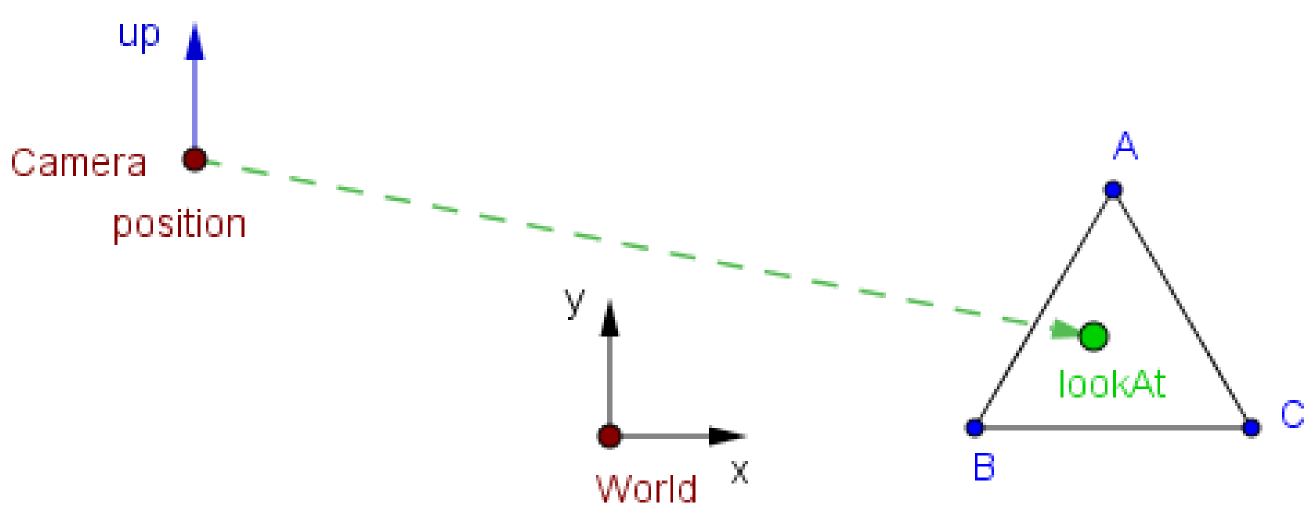
- Usually it is more intuitive to specify the camera by its **position**; **point** it is **looking at**; and the **up-vector**

**OpenGL:**

```
glm::mat4 view = glm::lookAt(
   glm::vec3(x, y, z),
   glm::vec3(pX, pY, pZ),
   glm::vec3(upX, upY, upZ)
);
```

# World Space → Camera Space

- Usually it is more intuitive to specify the camera by its **position**; **point** it is **looking at**; and the **up-vector**
- The up-vector may not be the same as the y-direction of camera's space. **It just gives a rough orientation.**



# World Space → Camera Space

- Using the lookAt() command parameters, how to find the view matrix?
- What do we have and what do we need?

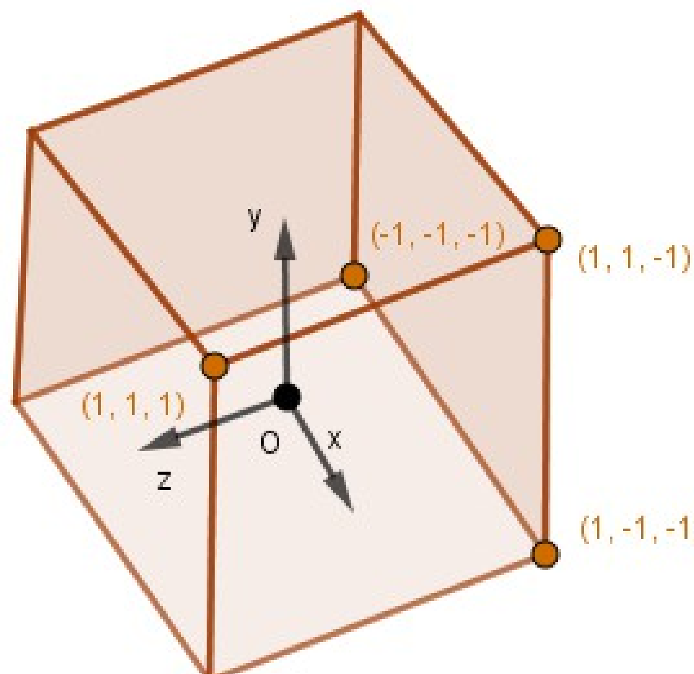
# World Space → Camera Space

$$\text{projectionMatrix} \cdot \text{viewMatrix} \cdot \text{modelMatrix} \cdot v$$

$$P \cdot V \cdot M \cdot v$$

# Camera Space → ND Space

- For the **normalized device space**, we transform the view frustum into a cube $[-1, 1]^3$.



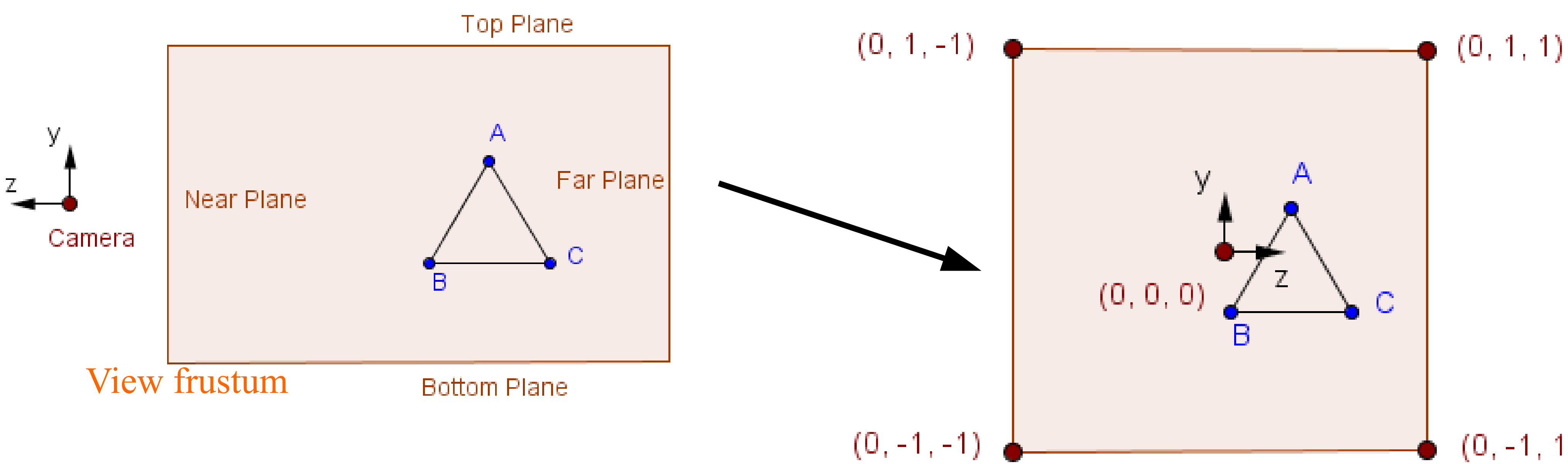

Canonical view volume

Frustum – *tüvipüramiid* (est)

# Camera Space → ND Space

- For the **normalized device space**, we transform the view frustum into a cube $[-1, 1]^3$.

**Orthographic**

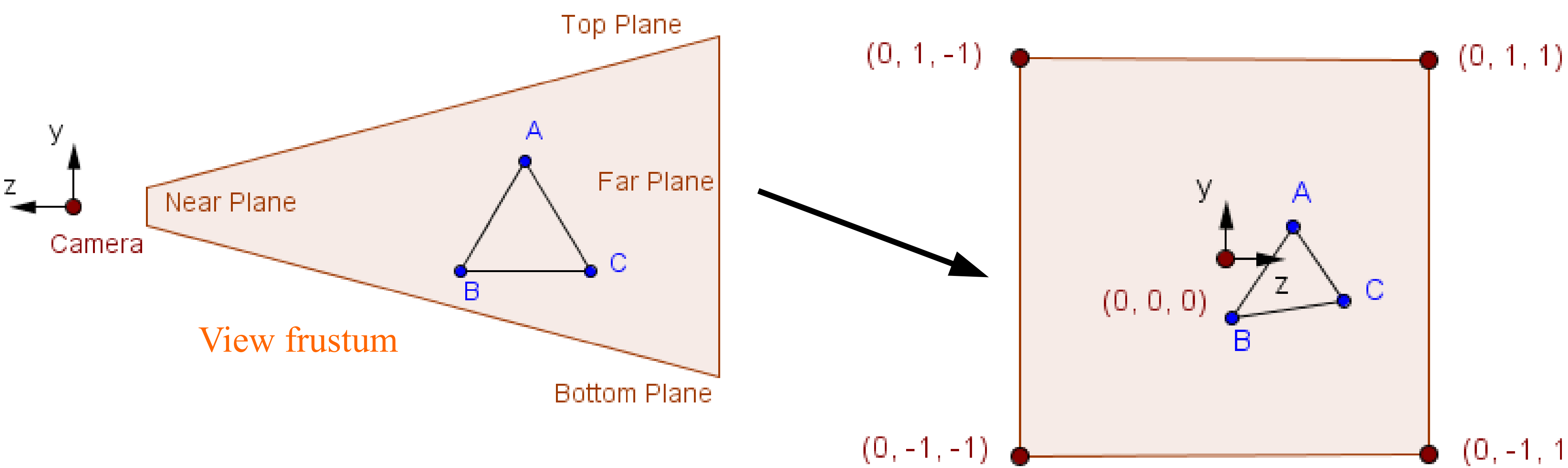



Slices from $x$=0 plane

# Camera Space → ND Space

- For the **normalized device space**, we transform the view frustum into a cube $[-1, 1]^3$.

**Perspective**




Slices from $x$=0 plane

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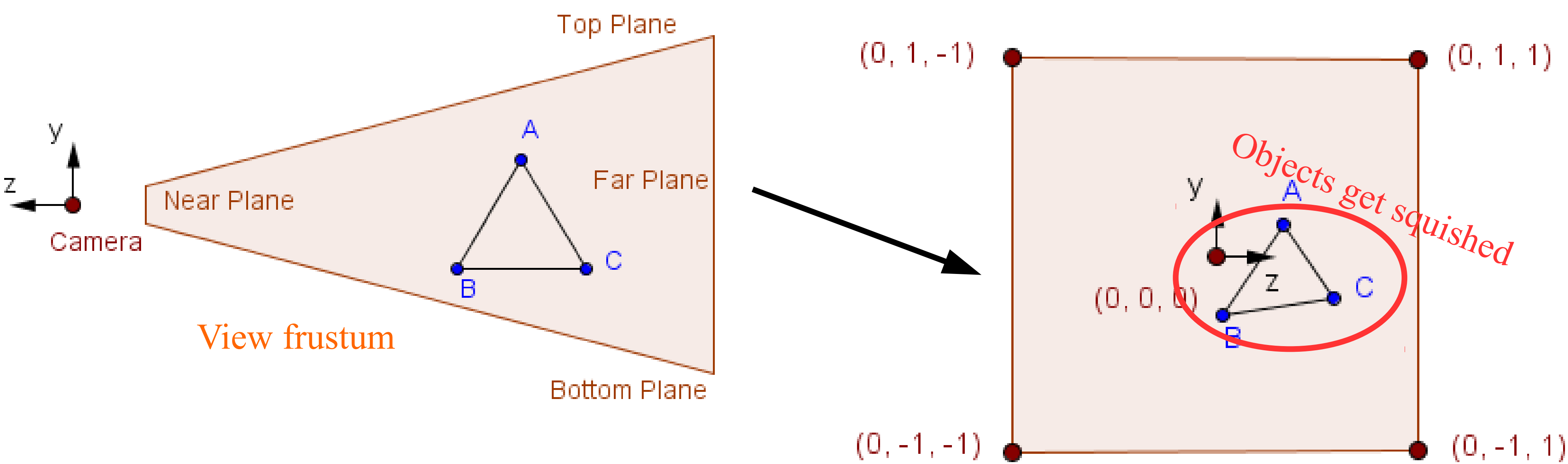
**Perspective**




See: http://tume-maailm.pri.ee/ylikool/CG/examples/perspectiveProjection/

# Camera Space → ND Space

- For the **normalized device space**, we transform the view frustum into a cube $[-1, 1]^3$.
- We want to flip the $z$-axis, because our near and far planes are positive values.

# Camera Space → ND Space

- For the **normalized device space**, we transform the view frustum into a cube $[-1, 1]^3$.
- We want to flip the z axis, because our near and far planes are positive values.
- This is the job for the **projection matrix** together with the **point normalization**.

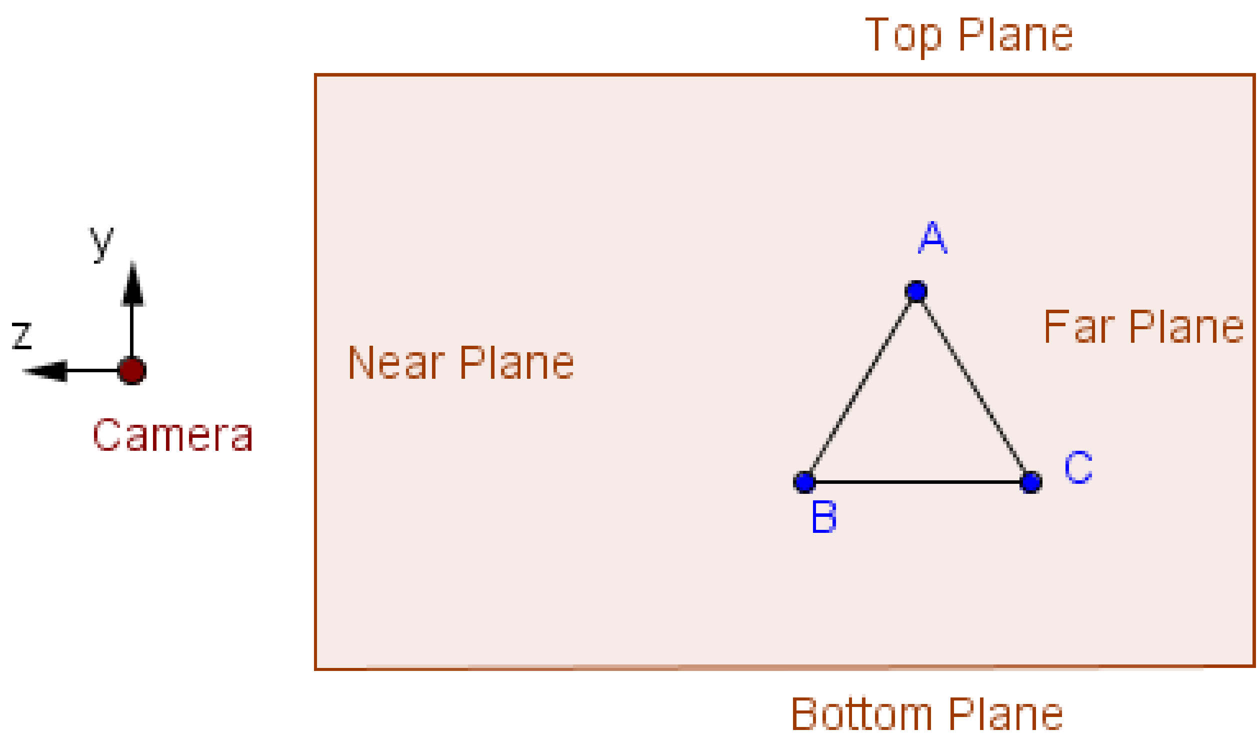
# Camera Space → ND Space

$$\text{projectionMatrix} \cdot \text{viewMatrix} \cdot \text{modelMatrix} \cdot v$$

$$P \cdot V \cdot M \cdot v$$

# Orthographic Projection

- We define our view volume with the values for **left**, **right**, **top**, **bottom**, **near** and **far** planes.



# Orthographic Projection

- We define our view volume with the values for **left**, **right**, **top**, **bottom**, **near** and **far** planes.

```
OrthographicCamera( left, right, top, bottom, near, far )

left — Camera frustum left plane.
right — Camera frustum right plane.
top — Camera frustum top plane.
bottom — Camera frustum bottom plane.
near — Camera frustum near plane.
far — Camera frustum far plane.

Together these define the camera's viewing frustum.
```

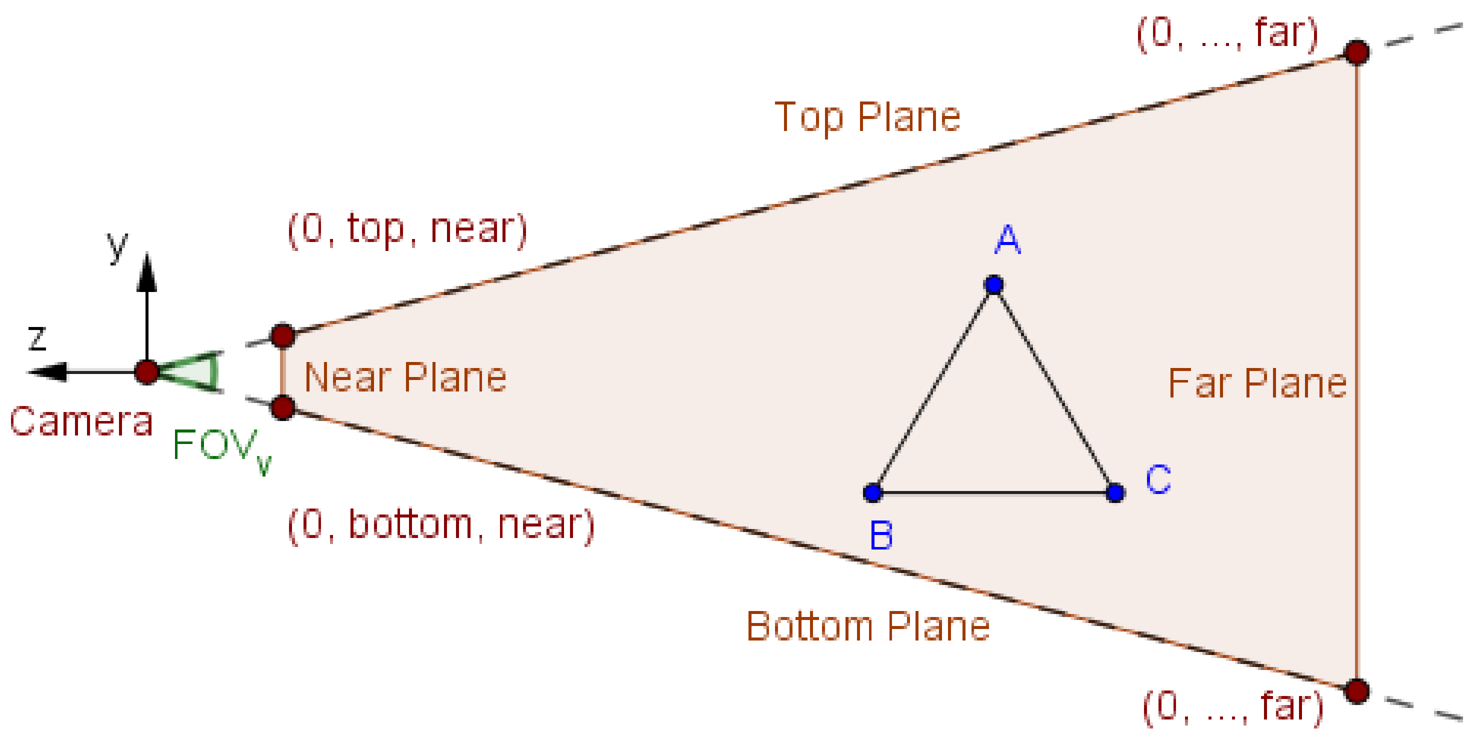
From Three.js docs.

# Orthographic Projection

- We define our view volume with the values for **left**, **right**, **top**, **bottom**, **near** and **far** planes.
- What would be the matrix that transforms the *orthographic view volume* into the *canonical view volume* ($[-1, 1]^3$)?

# Perspective Projection

- Usually defined by the **vertical angle** for the field-of-view (**FOV**), the **aspect ratio** and the **near** and **far** planes.



# Perspective Projection

- Usually defined by the **vertical angle** for the field-of-view (**FOV**), the **aspect ratio** and the **near** and **far** planes.

PerspectiveCamera( fov, aspect, near, far )

fov — Camera frustum vertical field of view.

aspect — Camera frustum aspect ratio.

near — Camera frustum near plane.

far — Camera frustum far plane.

Together these define the camera's viewing frustum.

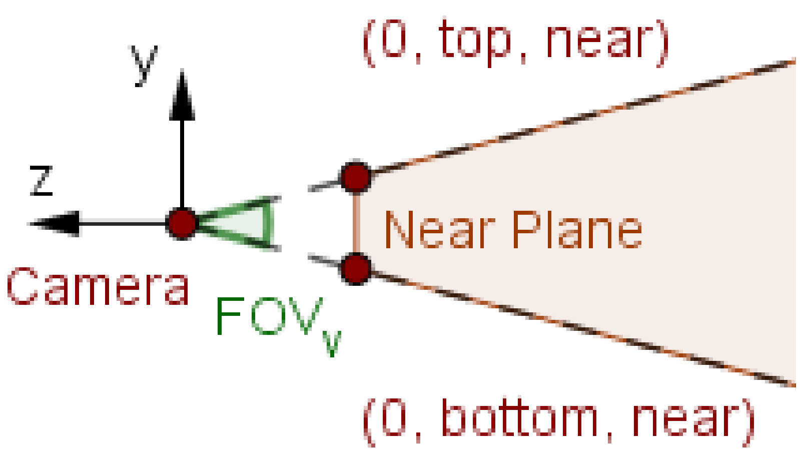
From Three.js docs.

# Perspective Projection

- Usually defined by the vertical angle for the field-of-view (**FOV**), the **aspect ratio** and the **near** and **far** planes.
- Find the ***left***, ***right***, ***top*** and ***bottom*** on the near plane, when the projection is *symmetric*?

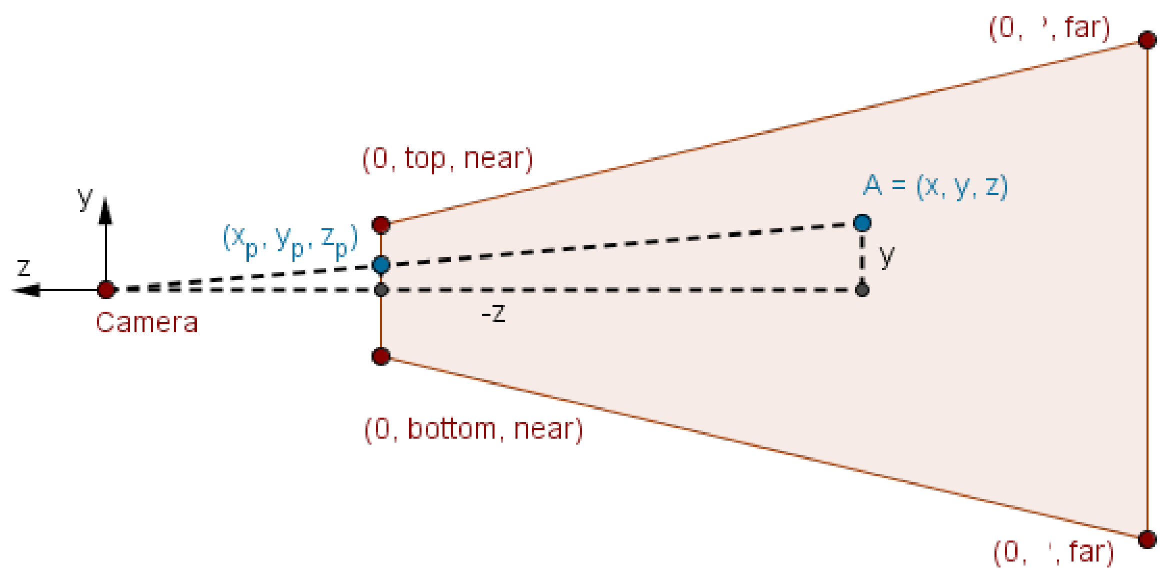
*top* = -*bottom*

*left* = -*right*



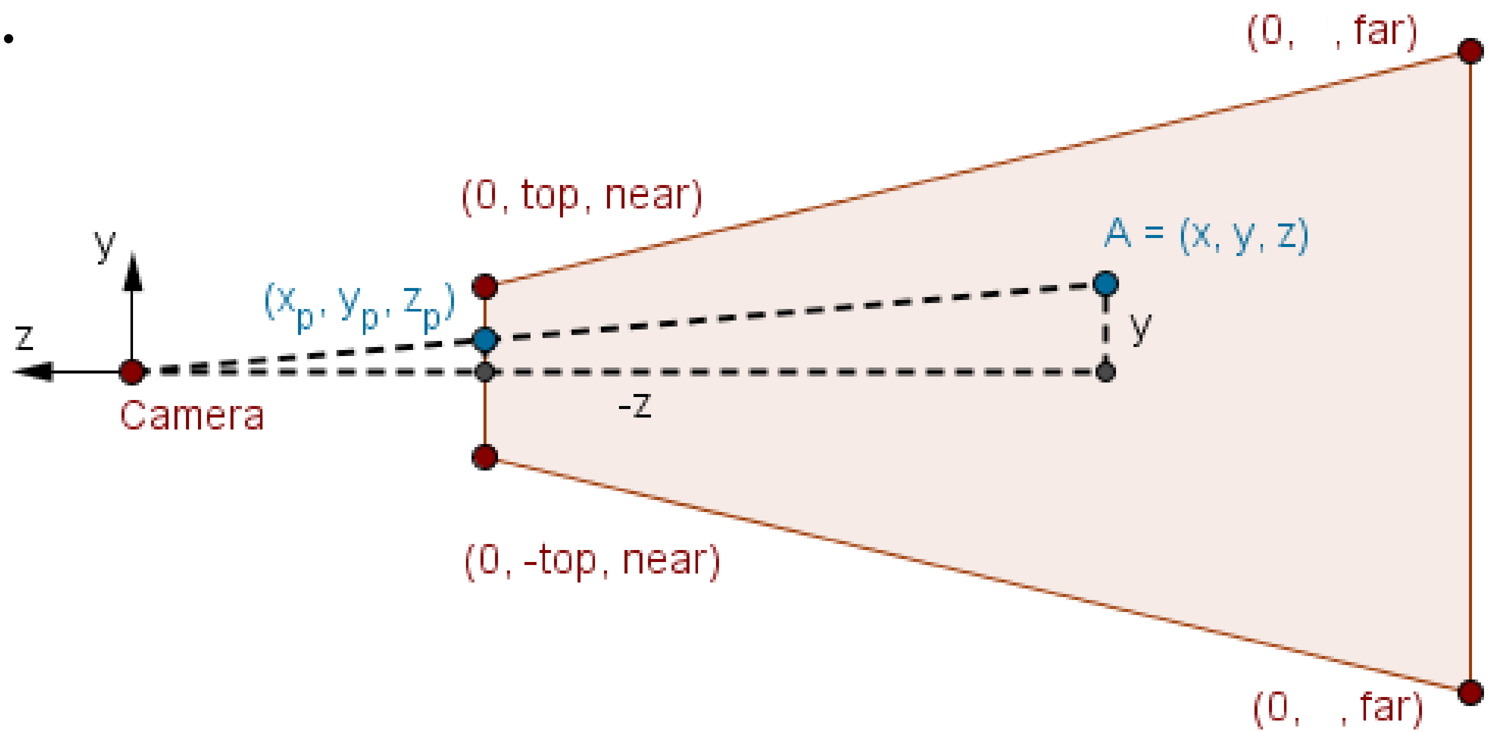

# Perspective Projection

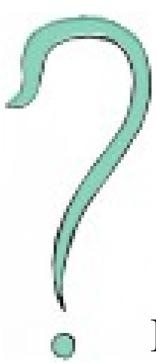
- Differently from the orthographic projection, here we have a viewer located in a single point.
- Similarly we want to find the normalized device coordinates for all points inside the view volume.



# Perspective Projection

- First **find** and then **map** the x and y coordinates of the ***projected point*** to the correct range using similar triangles.

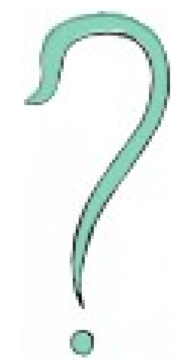


Find $x_p$ and $y_p$

# Perspective Projection

- First **find** and then **map** the x and y coordinates of the ***projected point*** to the correct range using similar triangles.
- Next map the values to the [-1, 1] range.



# Perspective Projection

- First **find** and then **map** the x and y coordinates of the ***projected point*** to the correct range using similar triangles.
- Next map the values to the [-1, 1] range.
- Lastly,
  - Take out the scaling parts for the matrix's diagonal.
  - Move the division with $z$ to the homogeneous division by changing the last row of the matrix.

# Perspective Projection

$$P = \begin{pmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

- If the third row is (0, 0, 1, 0), then all $z$ coordinates become -1 (beacuse we found the projected coordinates on the near plane)

# Perspective Projection

- We want to map the $z$ value from the range [near, far] to the range [-1, 1].
- We can use scale and translation.

$$P = \begin{pmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & s & t \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Perspective Projection

- We want to map the *z* value from the range [near, far] to the range [-1, 1], so...

$$\begin{cases} \dfrac{s \cdot near + t}{near} = -1 \\[2ex] \dfrac{s \cdot far + t}{far} = 1 \end{cases}$$

$$P = \begin{pmatrix} \dfrac{near}{right} & 0 & 0 & 0 \\[2ex] 0 & \dfrac{near}{top} & 0 & 0 \\[2ex] 0 & 0 & s & t \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Can this be solved for the *s* and *t* unknowns?

Division by *near* and *far* is because of the homogeneous division.

Hint

# Perspective Projection

- Applying this matrix and doing the point normalization (dividing with $w$), you get the perspective projection.

$$P = \begin{pmatrix} \frac{near}{right} & 0 & 0 & 0 \\ 0 & \frac{near}{top} & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2 \cdot far \cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Perspective Projection

- When using the FOV ($\alpha$) and aspect ratio ($ar$).

$$P = \begin{pmatrix} \frac{1}{ar \cdot \tan\left(\alpha/2\right)} & 0 & 0 & 0 \\ 0 & \frac{1}{\tan\left(\alpha/2\right)} & 0 & 0 \\ 0 & 0 & -\frac{far + near}{far - near} & -\frac{2 \cdot far \cdot near}{far - near} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

# Clip Space

- After the projection matrix multiplication and before the $w$-division, vertices are in a *clip space*.

Read more here: https://stackoverflow.com/a/21841924/3067608

# Clip Space

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- That is the space, where it is the most easiest to determine, which triangles need to be clipped or culled.

Read more here: https://stackoverflow.com/a/21841924/3067608

# Clip Space

- After the projection matrix multiplication and before the *w*-division, vertices are in a *clip space*.
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- **Clipping** – performed when some part of the triangle is inside the view volume.

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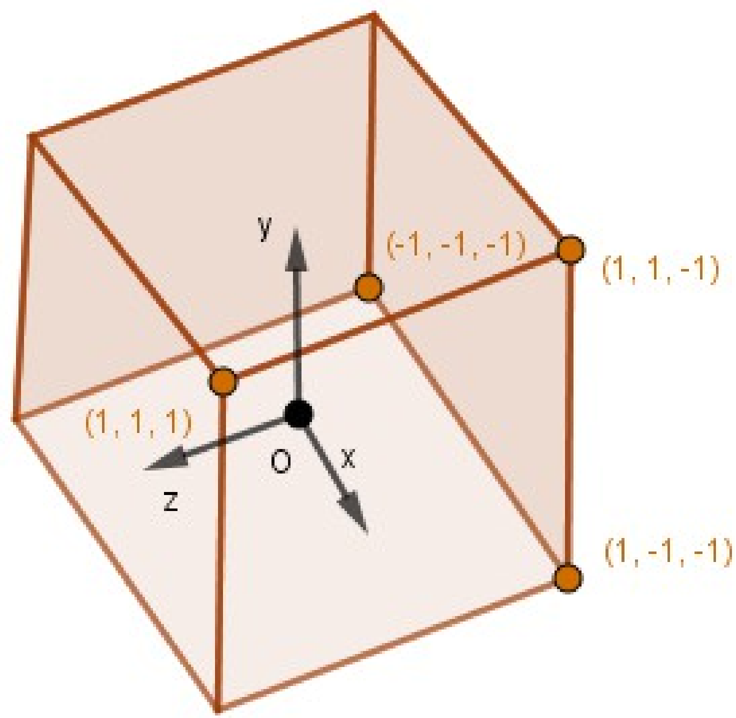
# Clip Space

- After the projection matrix multiplication and before the *w*-division, vertices are in a *clip space*.
- That is the space, where it is the most easiest to determine, which triangles need to be clipped or culled.
- **Clipping** – performed when some part of the triangle is inside the view volume.
- **Culling** – performed when the triangle is not inside the view volume. Or is back-facing.

Read more here: https://stackoverflow.com/a/21841924/3067608

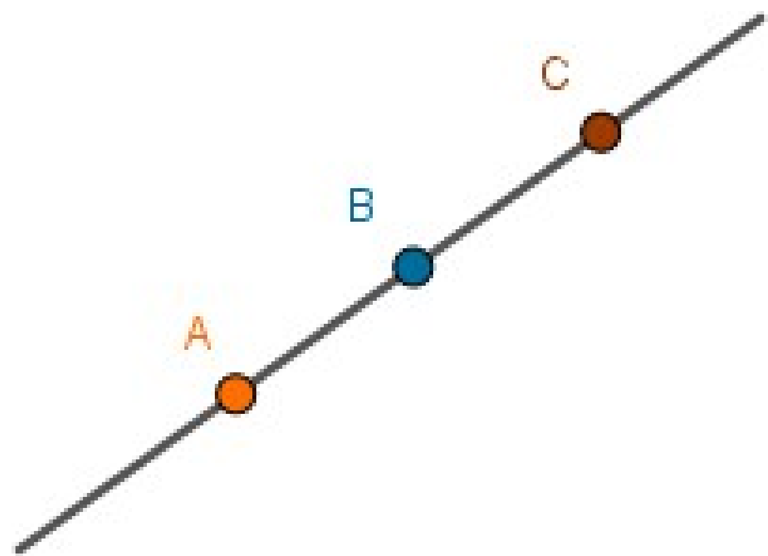
# ND Space → Screen Space

- We have everthing we want to show now in the $[-1, 1]^3$ cube (normalized device space).



# ND Space → Screen Space

- We have everthing we want to show now in the $[-1, 1]^3$ cube (normalized device space).
- We also know the correct relative depth of the vertices.




Before the perspective projection

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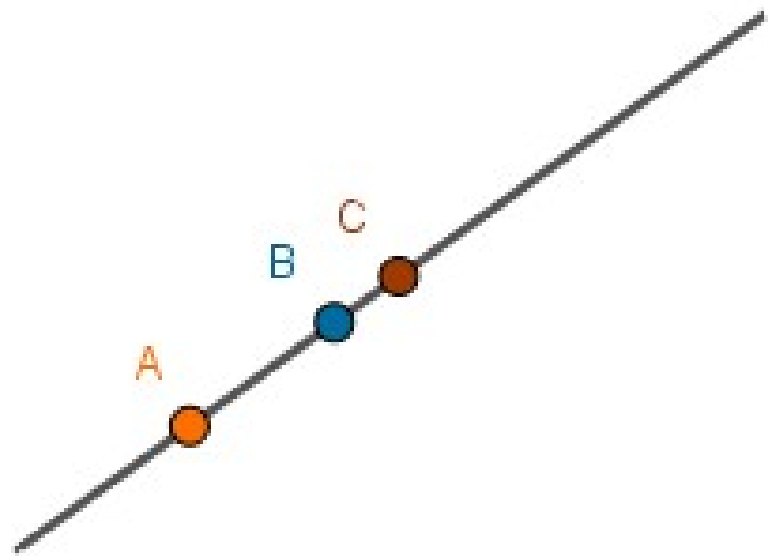
- We also know the correct relative depth of the vertices.




After the perspective projection

# ND Space → Screen Space

- We have everthing we want to show now in the $[-1, 1]^3$ cube (normalized device space).
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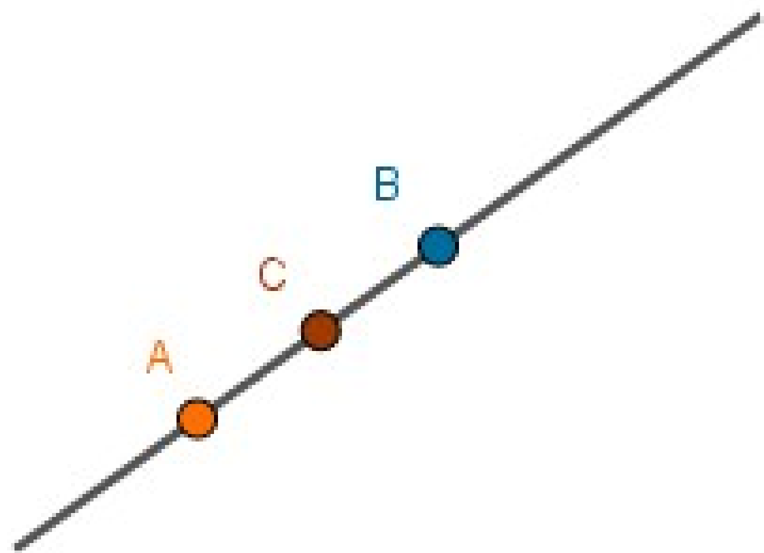



This will not happen!

# ND Space → Screen Space

- We have everthing we want to show now in the $[-1, 1]^3$ cube (normalized device space).
- We also know the correct relative depth of the vertices.
- How to know where to draw on the screen?

Come up with that matrix...

# ND Space → Screen Space

- This is done for you, the screen space matrix is constructed when you specify the viewport size.

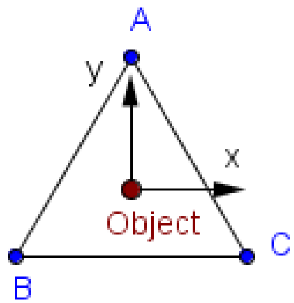
**Three.js**

```
renderer = new THREE.WebGLRenderer();
renderer.setSize(width, height);
```

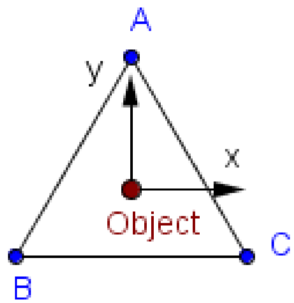
**OpenGL + GLFW**

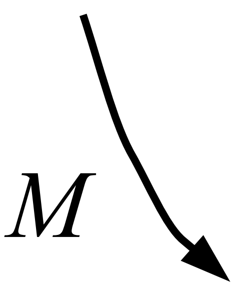
```
win = glfwCreateWindow(width, height,
                       "Hello GLFW!", NULL, NULL)
```

# Object Space

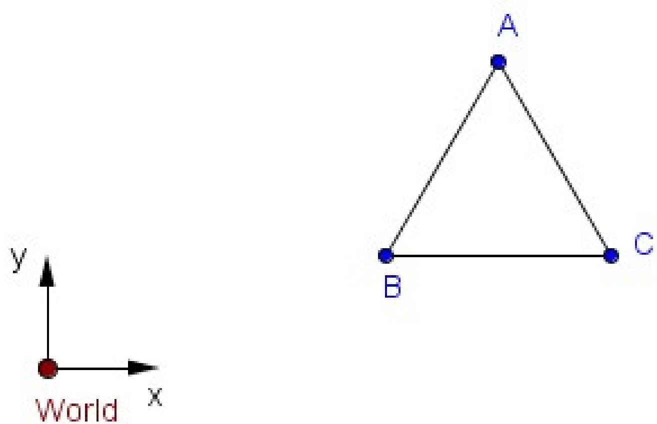


# Object Space

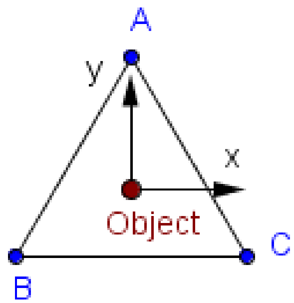



# World Space

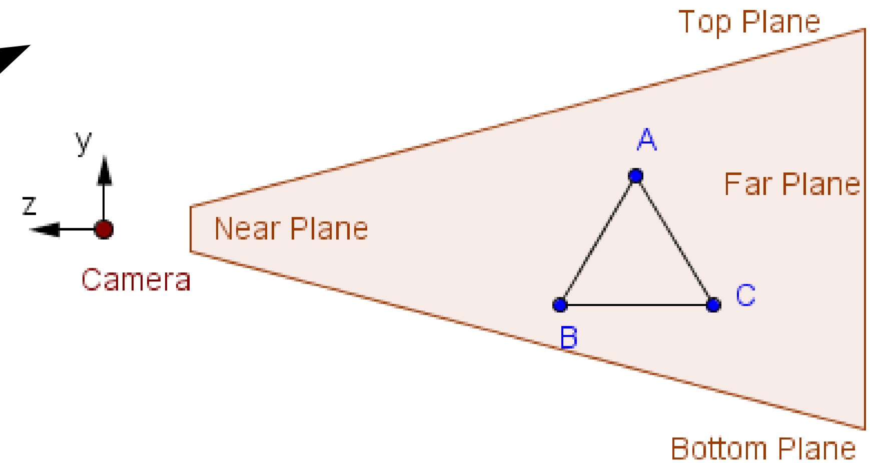


# Object Space



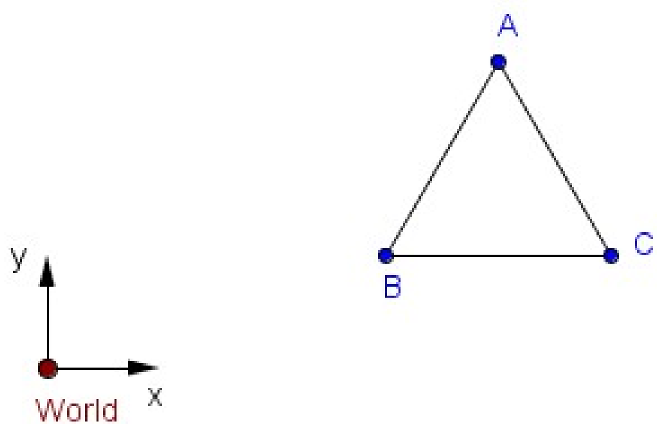

$M$

# Camera (View) Space

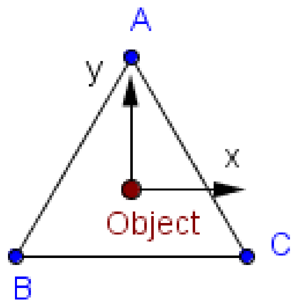


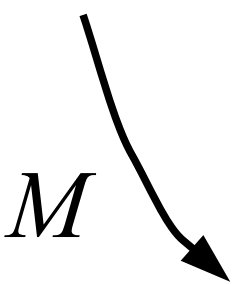

$V$

# World Space

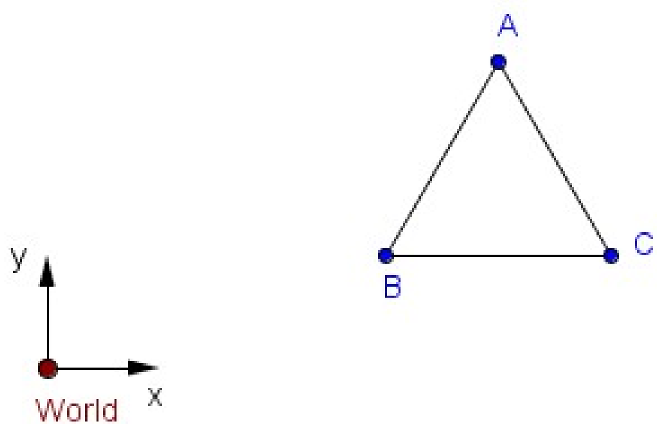


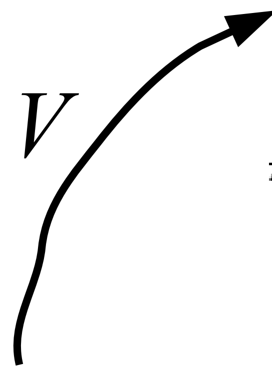
# Object Space



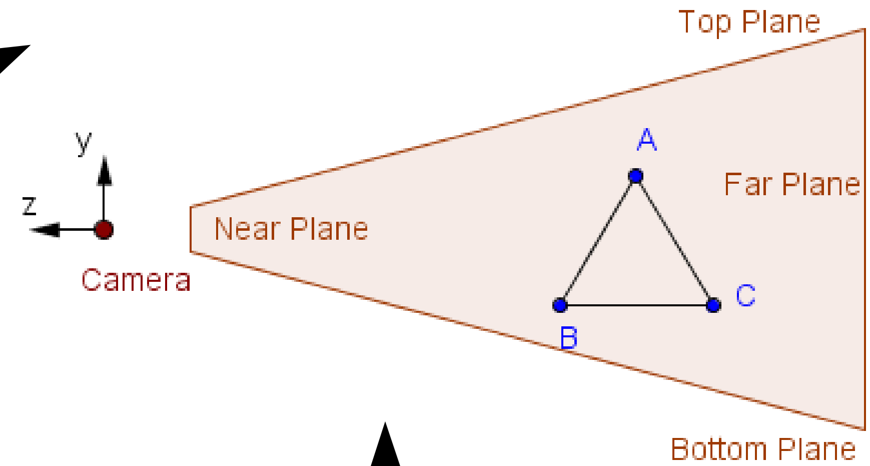




# World Space



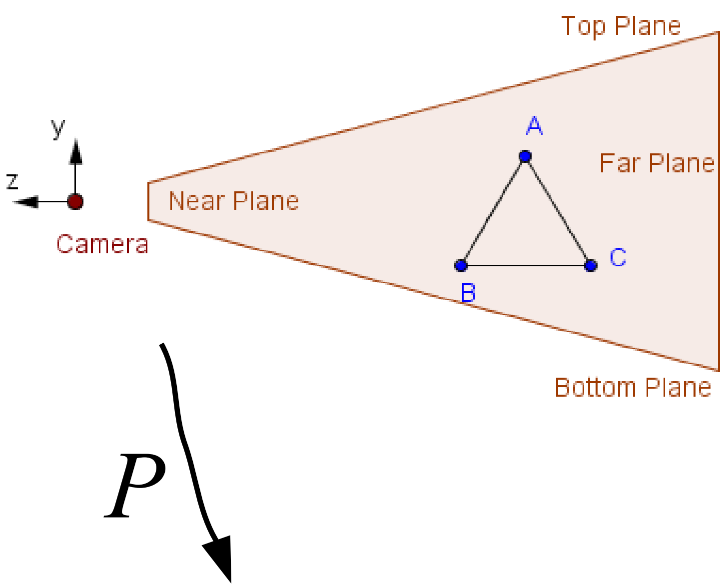




# Camera (View) Space



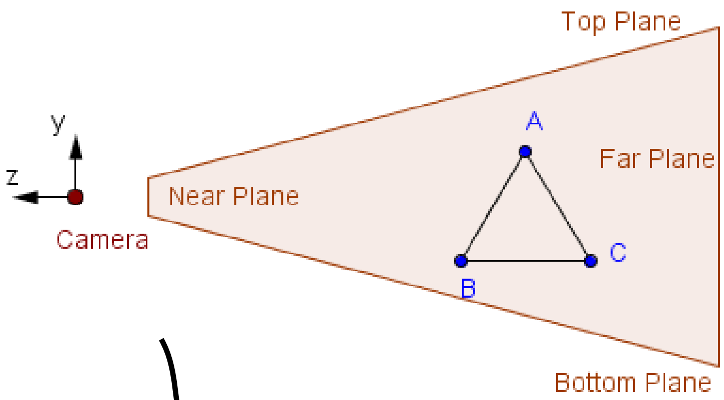

Light calculations are usually in this space!

# Camera (View) Space




# Clip Space

# Camera (View) Space

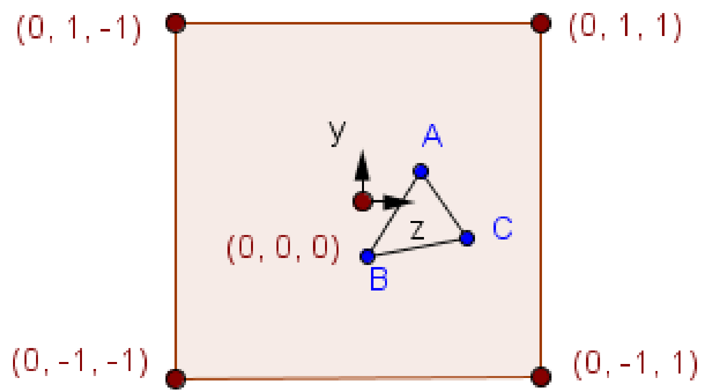



$P$

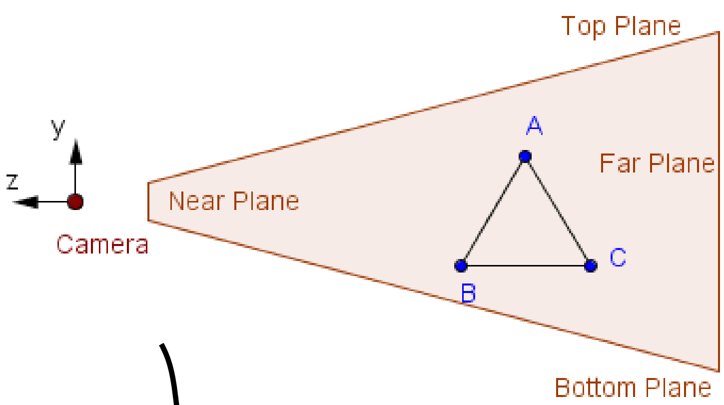
# Clip Space

$$\left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)$$

# Normalized Device Space



# Camera (View) Space

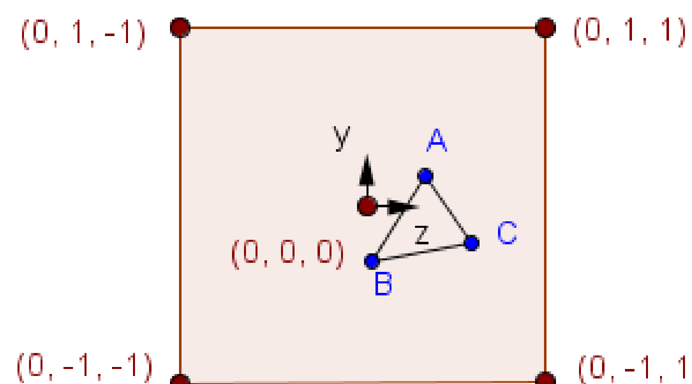



$P$

# Clip Space

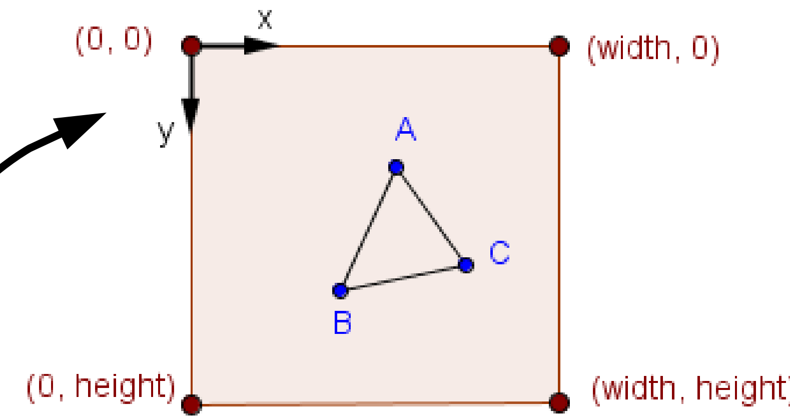
$$\left(\frac{x}{w}, \frac{y}{w}, \frac{z}{w}\right)$$

# Normalized Device Space




$S$

# Sceen Space



# Vertex Shader

- Vertex shader must return homogeneous coordinates in the clip space – that is in normalized device space without the *w*-division.

```
gl_Position = projection * view * model * vec4(position, 1.0);

gl_Position = projectionMatrix * modelViewMatrix * vec4(position, 1.0);

gl_Position = modelViewProjectionMatrix * vec4(position, 1.0);
```

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```

- Then GPU does:
  - *w*-division
  - Screen space transformation

# Additional Links

- General overview: http://www.opengl-tutorial.org/beginners-tutorials/tutorial-3-matrices/
- How to derive the view matrix: http://3dgep.com/understanding-the-view-matrix/
- How to derive the projection matrices: http://www.songho.ca/opengl/gl_projectionmatrix.html
- About transforming the surface normals: http://www.lighthouse3d.com/tutorials/glsl-tutorial/the-normal-matrix/

What was interesting for you today?

What more would you like to know?

Next time:
Shading and Lighting