Computer Graphics Seminar

MTAT.03.305

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Computer Graphics

- Graphical illusion via the computer
- Displaying something meaningful (inc art)
Math

- Computers are good at... computing.
- To do computer graphics, we need math for the computer to compute.
- Geometry, algebra, calculus.

\[
\begin{bmatrix}
a & b & c & d \\
e & f & g & h \\
i & j & k & l \\
0 & 0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
ax + by + cz + d \\
ex + fy + gz + h \\
ix + jy + kz + l \\
1
\end{bmatrix}
\]
Point

- Simplest geometry primitive
- In homogeneous coordinates: \((x, y, z, w), \ w \neq 0\)

\[(x, y, z, 1)\]

- Represents a point \((x/w, y/w, z/w)\)
- Usually you can put \(w = 1\)
- Actual division will be done by GPU later
Line (segment)

- Consists of:
  - 2 endpoints
  - \textit{Infinite} number of points between
- Defined by the endpoints
- Interpolated in GPU

\begin{align*}
  (x_1, y_1, z_1, 1) \\
  (x_2, y_2, z_2, 1)
\end{align*}
Triangle

- Consists of:
  - 3 points called vertices
  - 3 lines called edges
  - 1 face

- Defined by 3 vertices

- Lines and face will be interpolated in GPU

- Counter-clockwise order defines front face
Why triangles?

• They are in many ways the simplest polygons
  • 3 different points always form a plane
  • Easy to rasterize (fill the face with points)
  • Every other polygon can be converted to triangles

• OpenGL used to support other polygons too
  • Must have been:
    – Simple – No edges intersect each other
    – Convex – All points between any two points are inner points
Examples of polygons

1. Pentagon
2. Hexagon
3. Diamond
4. Triangle
OpenGL < 3.1 primitives

Figure 2-7  Geometric Primitive Types

OpenGL Programming Guide 7th edition, p49
After OpenGL 3.1

Table 3.1  OpenGL Primitive Mode Tokens

<table>
<thead>
<tr>
<th>Primitive Type</th>
<th>OpenGL Token</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points</td>
<td>GL_POINTS</td>
</tr>
<tr>
<td>Lines</td>
<td>GL_LINES</td>
</tr>
<tr>
<td>Line Strips</td>
<td>GL_LINE_STRIP</td>
</tr>
<tr>
<td>Line Loops</td>
<td>GL_LINE_LOOP</td>
</tr>
<tr>
<td>Independent Triangles</td>
<td>GL_TRIANGLES</td>
</tr>
<tr>
<td>Triangle Strips</td>
<td>GL_TRIANGLE_STRIP</td>
</tr>
<tr>
<td>Triangle Fans</td>
<td>GL_TRIANGLE_FAN</td>
</tr>
</tbody>
</table>

Figure 3.1  Vertex layout for a triangle strip

Figure 3.2  Vertex layout for a triangle fan

In the beginning there were points

- Now that we can define our geometric objects, what next?
- We want to move our objects!

Luckily GPU will do this work for us.
Transformations

It turns out that homogeneous coordinates allows us to easily do:

- Linear transformations
  - Scaling, reflection
  - Rotation
  - Shearing
- Affine transformations
  - Translation (shifting)
- Projection transformations
  - Perspective
  - Orthographic

Actually these we could do without homogeneous coordinates...

This too...
Transformations

- Every transformation is a function
- As you remember from Algebra, linear functions can be represented as matrices

\[
f(v) = (2x, y, z) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}
\]

\[v \in \mathbb{R}^3\]
\[v = (x, y, z)\]

You should transpose the result later
Transformations

- GPU-s are built for doing transformations with matrices.
- Remember, computers are made for computing.
- Let's look at some linear transformations...

\[ f(a_1 x_1 + \ldots + a_n x_n) = a_1 f(x_1) + \ldots + a_n f(x_n) \]

We don't use homogeneous coordinates at the moment, don't worry, they'll be back...
Scaling

- Redefines the basis vectors as some multiple of the previous basis vectors.

\[
\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.5 \end{pmatrix}
\]
Scaling

• In general we could scale each axis

\[
\begin{pmatrix}
a_x & 0 & 0 \\
0 & a_y & 0 \\
0 & 0 & a_z
\end{pmatrix}
\]

\(a_x\) – x-axis scale factor

\(a_y\) – y-axis scale factor

\(a_z\) – z-axis scale factor

• If some factor is negative, this matrix will reflect the points from that axis. Thus we get reflection.
Shearing

- Not much used by itself, but remember it for translations later.
- Tilts only one axis.
- Squares become parallelograms.

\[
\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}
\]

\[
\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}
\]
Shearing

- **Shear-x**, we tilt $x$ basis vector by angle $\phi$ counter-clockwise

\[
\begin{pmatrix}
1 & 0 \\
\tan(\phi) & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
x \\
y + \tan(\phi) \cdot x
\end{pmatrix}
\]

- **Shear-y**, we tilt $y$ basis vector by angle $\phi$ clockwise

\[
\begin{pmatrix}
1 & \tan(\phi) \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
x + \tan(\phi) \cdot y \\
y
\end{pmatrix}
\]
Rotation

- Shearing rotated only one axis
- Also changed the size of the basis vector
- Can we do better?

Did you notice that the columns of the transformation matrix show the coordinates of the new basis vectors?
Rotation

\[ e'_0 = (|a|, |b|) = (\cos(\alpha), \sin(\alpha)) \]
\[ e'_1 = (|a'|, |b'|) = (-\sin(\alpha), \cos(\alpha)) \]

\[ \cos(\alpha) = \frac{|a|}{|e'_0|} = \frac{|a|}{1} = |a| \]
Rotation

- So if we rotate by $\alpha$ in counter-clockwise order in 2D, the transformation matrix is:

$$
\begin{pmatrix}
\cos(\alpha) & -\sin(\alpha) \\
\sin(\alpha) & \cos(\alpha)
\end{pmatrix}
$$

- In 3D we can do rotations in each plane (xy, xz, yz), so there can be 3 different matrices.
Rotation

- To do a rotation around an arbitrary axis, we can:
  - Rotate that axis to be the x-axis
  - Rotate around the new x-axis
  - Invert the first rotations (move the old x-axis back)
  - OpenGL provides a command for rotating around a given axis.
  - Sometimes quaternions are used for rotations.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\alpha) & -\sin(\alpha) & 0 \\
0 & \sin(\alpha) & \cos(\alpha) & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

Quaternions are elements of a number system that extend the complex numbers...
Do we have everything now?

- We can scale, share and rotate our geometry around the origin...

What if we have an object not centered in the origin?
Translation

• Imagine that our 1D world is located at y=1 line in 2D space.

• Notice that all the points are in the form: \((x, 1)\)
Translation

- What happens if we do shear-y(45°) operation on the 2D world?

- Everything in our world has moved magically one x-coordinate to the right...

\[ \tan(45°) = 1 \]
Translation

- What if we do shear-\(y(63.4^\circ)\)?

\[
\tan(63.4^\circ) = 2
\]

- Everything has now moved 2 x-coordinates to the right from the original position

- We can do translation / shift!
Translation

- When we represent our points in one dimension higher space, where the extra coordinate is 1, we get to the **homogeneous** space.

$\begin{pmatrix} 1 & x_t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ z + z_t \\ 1 \end{pmatrix}$
Transformations

- This together gives us a very good toolset to transform our geometry as we wish.

\[
\begin{pmatrix}
a & b & c & x_t \\
d & e & f & y_t \\
g & h & i & z_t \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
1
\end{pmatrix}
= 
\begin{pmatrix}
ax + by + cz + x_t \\
dx + ey + fz + y_t \\
gx + hy + iz + z_t \\
1
\end{pmatrix}
\]
Multiple transformations

- Everything starts from the origin!
- To apply multiple transformations, just multiply matrices.
Multiple transformations

Our initial geometry defined by vertices: (-1, -1), (1, -1), (1, 1), (-1, 1)
Multiple transformations

\[
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) & 0 \\
\sin(45^\circ) & \cos(45^\circ) & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
Multiple transformations

\[
\begin{pmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
Multiple transformations

- We can combine the transformations to a single matrix.

\[
\begin{pmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) & 0 \\
\sin(45^\circ) & \cos(45^\circ) & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
= 
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) & 4 \\
\sin(45^\circ) & \cos(45^\circ) & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

- This also works for combining different linear transformations, but the resulting matrix isn't that clear...

- Order of transformations / matrices is important!

- [http://cgdemos.tume-maailm.pri.ee](http://cgdemos.tume-maailm.pri.ee)
Now we know how it's supposed to go...
OpenGL

- GPU API / middleware
- I.e a set of commands that program can give to GPU
- Supported in many languages
WebGL

- GPU API in JavaScript
- Supported by major browsers
- THREE.js – Higher level library to ease your coding: [http://threejs.org/docs/](http://threejs.org/docs/)
Forward is backward

• Actually you need to tell GPU commands in this order:
  • Transformations
  • Vertices

```
float glTranslatef(float x, float y, float z) { ... }
float glRotatef(float angle, float x, float y, float z) { ... }

float glBegin(GL_QUADS) { ... }
float glVertex3fv(float x, float y, float z) { ... }
float glEnd() { ... }
```
State machine

- GPU acts like a state machine

GPU is now in a state to receive quad vertices

Sending vertices to GPU

Stop being in that state

Prior to OpenGL 3
Multiple objects in the scene?

- Each object has its own geometry
- And its own transformations for that geometry
- OpenGL has a single ModelView matrix

```
glTranslatef(0.0, 0.0, -1.0);
glRotatef(60, 1.0, 0.0, 0.0);
glRotatef(-20, 0.0, 0.0, 1.0);
glBegin(GL_QUADS);
  glVertex3fv(...);
  glVertex3fv(...);
  glVertex3fv(...);
  glVertex3fv(...);
glEnd();
```
Multiple objects in the scene

- OpenGL has a matrix stack
- We can push a copy to the stack (save)
- We can pop the top matrix from the stack (load)

Prior to OpenGL 3
Multiple objects in the scene

- More complex geometry for a single object

Prior to OpenGL 3
Old and new OpenGL?

- Lot has changed from OpenGL 3.
- Everything old still works in compatibility mode.

<table>
<thead>
<tr>
<th>Prior to OpenGL 3</th>
<th>OpenGL 3+</th>
</tr>
</thead>
<tbody>
<tr>
<td>glBegin(...)</td>
<td>Vertex Array Object (VAO)</td>
</tr>
<tr>
<td>glVertex(...)</td>
<td>Vertex Buffer Object (VBO)</td>
</tr>
<tr>
<td>glEnd(...)</td>
<td>Use other Matrix library</td>
</tr>
<tr>
<td>glTranslate(...)</td>
<td>Send your matrices to shaders</td>
</tr>
<tr>
<td>glRotate(...)</td>
<td>Vertex Buffer Object</td>
</tr>
<tr>
<td>glScale(...)</td>
<td>Vertex Buffer Object</td>
</tr>
<tr>
<td>glMaterial(...)</td>
<td></td>
</tr>
</tbody>
</table>
Multiple objects in the scene

- Useful to think of the scene as a tree
Next time...

- Graphics pipeline in more detail
- How to define color for our geometry?
- Vertex and fragment shaders
- What else could be done?

Our geometry defined by 4 vertices
A parallelogram...