Computer Graphics

- Graphical illusion via the computer
- Displaying something meaningful (inc art)
Math

- Computers are good at... computing.
- To do computer graphics, we need math for the computer to compute.
- Geometry, algebra, calculus.

\[
\begin{bmatrix}
a & b & c & d \\
e & f & g & h \\
i & j & k & l \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
=
\begin{bmatrix}
ax + by + cz + d \\
ex + fy + gz + h \\
ix + jy + kz + l \\
1
\end{bmatrix}
\]
Point

• Simplest geometry primitive
• In homogeneous coordinates: \((x, y, z, w), w \neq 0\)

\((x, y, z, 1)\)

• Represents a point \((x/w, y/w, z/w)\)
• Usually you can put \(w = 1\)
• Actual division will be done by GPU later
Line (segment)

- Consists of:
  - 2 endpoints
  - *Infinite* number of points between
- Defined by the endpoints
- Interpolated and rasterized in the GPU

\((x_1, y_1, z_1, 1)\)

\((x_2, y_2, z_2, 1)\)
Triangle

- Consists of:
  - 3 points called vertices
  - 3 lines called edges
  - 1 face
- Defined by 3 vertices
- Face interpolated and rasterized in the GPU
- Counter-clockwise order defines front face
Why triangles?

- They are in many ways the simplest polygons
  - 3 different points always form a plane
  - Easy to rasterize (fill the face with pixels)
  - Every other polygon can be converted to triangles
- OpenGL used to support other polygons too
  - Must have been:
    - Simple – No edges intersect each other
    - Convex – All points between any two points are inner points
Examples of polygons
OpenGL < 3.1 primitives

Figure 2-7  Geometric Primitive Types

OpenGL Programming Guide 7th edition, p49
After OpenGL 3.1

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**Figure 3.1**  Vertex layout for a triangle strip

**Figure 3.2**  Vertex layout for a triangle fan

In the beginning there were points

- Now that we can define our geometric objects, what next?
- We want to move our objects!

Luckily GPU will do this work for us.
Transformations

• It turns out that homogeneous coordinates allows us to easily do:
  • Linear transformations
    – Scaling, reflection
    – Rotation
    – Shearing
  • Affine transformations
    – Translation (moving / shifting)
  • Projection transformations
    – Perspective
    – Orthographic
Transformations

- Every transformation is a function
- As you remember from Algebra, linear functions can be represented as matrices

\[
f(v) = (2 \cdot x, y, z) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}
\]

\(v \in \mathbb{R}^3\)

\(v = (x, y, z)\)

You should transpose the result later
Transformations

- GPU-s are built for doing transformations with matrices on points (vertices).
- Remember, computers are made for computing.
- Let's look at some linear transformations...

$$f(a_1 x_1 + \ldots + a_n x_n) = a_1 f(x_1) + \ldots + a_n f(x_n)$$

We don't use homogeneous coordinates at the moment, don't worry, they'll be back...
Scaling

- Redefines the basis vectors as some multiple of the previous basis vectors.

\[
\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.5 \end{pmatrix}
\]

These coordinates are in the old basis.
Scaling

• In general we could scale each axis

\[
\begin{pmatrix}
  a_x & 0 & 0 \\
  0 & a_y & 0 \\
  0 & 0 & a_z
\end{pmatrix}
\]

\(a_x\) – x-axis scale factor

\(a_y\) – y-axis scale factor

\(a_z\) – z-axis scale factor

• If some factor is negative, this matrix will reflect the points from that axis. Thus we get reflection.
Shearing

- Not much used by itself, but remember it for translations later.
- Tilts only one axis.
- Squares become parallelograms.

\[
\begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix} \cdot \begin{pmatrix}
0 \\
2
\end{pmatrix} = \begin{pmatrix}
0 \\
2
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix} \cdot \begin{pmatrix}
1 \\
2
\end{pmatrix} = \begin{pmatrix}
1 \\
3
\end{pmatrix}
\]
Shearing

- **Shear-y**, we tilt parallel to y axis by angle $\phi$ counter-clockwise
  \[
  \begin{pmatrix} 1 & 0 \\ \tan(\phi) & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y + \tan(\phi) \cdot x \end{pmatrix}
  \]

- **Shear-x**, we tilt parallel to x axis by angle $\phi$ clockwise
  \[
  \begin{pmatrix} 1 & \tan(\phi) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \tan(\phi) \cdot y \\ y \end{pmatrix}
  \]
Rotation

- Shearing rotated only one axis
- Also changed the size of the basis vector
- Can we do better?

Did you notice that the columns of the transformation matrix show the coordinates of the new basis vectors?
\[ e'_0 = (|a|, |b|) = (\cos(\alpha), \sin(\alpha)) \]
\[ e'_1 = (|a'|, |b'|) = (-\sin(\alpha), \cos(\alpha)) \]

\[ \cos(\alpha) = \frac{|a|}{|e'_0|} = \frac{|a|}{1} = |a| \]
Rotation

- So if we rotate by $\alpha$ in counter-clockwise order in 2D, the transformation matrix is:

\[
\begin{pmatrix}
\cos(\alpha) & -\sin(\alpha) \\
\sin(\alpha) & \cos(\alpha)
\end{pmatrix}
\]

- In 3D we can do rotations in each plane (xy, xz, yz), so there can be 3 different matrices.
Rotation

• To do a rotation around an arbitrary axis, we can:
  • Rotate that axis to be the x-axis
  • Rotate around the new x-axis
  • Invert the first rotations (move the old x-axis back)

• OpenGL provides a command for rotating around a given axis.

• Often quaternions are used for rotations.

Quaternions are elements of a number system that extend the complex numbers...
Do we have everything now?

- We can scale, share and rotate our geometry around the origin...

What if we have an object not centered in the origin?
Translation

- Imagine that our 1D world is located at y=1 line in 2D space.

- Notice that all the points are in the form: \((x, 1)\)
Translation

• What happens if we do shear-x(45°) operation on the 2D world?

• Everything in our world has moved magically one x-coordinate to the right...

\[
\tan(45°) = 1
\]
Translation

- What if we do shear-x(63.4°)?
  \[ \tan(63.4°) = 2 \]

- Everything has now moved 2 x-coordinates to the right from the original position.
- We can do translation (movement)!
Translation

- When we represent our points in one dimension higher space, where the extra coordinate is 1, we get to the **homogeneous** space.

\[
\begin{pmatrix}
1 & x_t \\
0 & 1
\end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ 1 \end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & x_t \\
0 & 1 & y_t \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ 1 \end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 & x_t \\
0 & 1 & 0 & y_t \\
0 & 0 & 1 & z_t \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ z + z_t \\ 1 \end{pmatrix}
\]
Transformations

- This together gives us a very good toolset to transform our geometry as we wish.

\[
\begin{pmatrix}
a & b & c & x_t \\
d & e & f & y_t \\
g & h & i & z_t \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
1
\end{pmatrix} =
\begin{pmatrix}
ax + by + cz + x_t \\
dx + ey + fz + y_t \\
gx + hy + iz + z_t \\
1
\end{pmatrix}
\]

Used for perspective projection...
Multiple transformations

- Everything starts from the origin!
- To apply multiple transformations, just multiply matrices.
Multiple transformations

Our initial geometry defined by vertices: (-1, -1), (1, -1), (1, 1), (-1, 1)
Multiple transformations

\[
\begin{pmatrix}
\cos(45°) & -\sin(45°) & 0 \\
\sin(45°) & \cos(45°) & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
Multiple transformations

\[
\begin{pmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
Multiple transformations

- We can combine the transformations to a single matrix.

\[
\begin{pmatrix}
1 & 0 & 4 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) & 0 \\
\sin(45^\circ) & \cos(45^\circ) & 0 \\
0 & 0 & 1
\end{pmatrix} =
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) & 4 \\
\sin(45^\circ) & \cos(45^\circ) & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

- This also works for combining different linear transformations, but the resulting matrix isn't that clear...

- Order of transformations / matrices is important!

- [http://cgdemos.tume-maailm.pri.ee](http://cgdemos.tume-maailm.pri.ee)
Now we know how it's supposed to go...
OpenGL

- GPU API / middleware
- Set of commands that program can give to GPU
- Supported in many languages

Vertices a, b, c
Transformations A, B, C
Draw!
WebGL

- GPU API in JavaScript
- Supported by major browsers
- THREE.js – Higher level library to ease your coding: http://threejs.org/docs/

Vertices a, b, c

Transformations A, B, C

Draw!
Forward is backward

- Actually you need to tell GPU commands in this order:
  - Transformations
  - Vertices

```cpp
glTranslatef(0.0, 0.0, -1.0);
glRotatef(60, 1.0, 0.0, 0.0);
glRotatef(-20, 0.0, 0.0, 1.0);

glBegin(GL_QUADS);
glVertex3fv(...);
glVertex3fv(...);
glVertex3fv(...);
glVertex3fv(...);
glEnd();
```
State machine

• GPU acts like a state machine

Prior to OpenGL 3
Multiple objects in the scene?

- Each object has its own geometry
- And its own transformations for that geometry
- OpenGL had a single ModelView matrix

Prior to OpenGL 3
Multiple objects in the scene

- OpenGL had a matrix stack
- We can push a copy to the stack (save)
- We can pop the top matrix from the stack (load)

Prior to OpenGL 3
Multiple objects in the scene

- More complex geometry for a single object

```
MV

MV*A

Saved matrices here

MV*A*B

MV*A

MV

glPushMatrix();
glTranslatef(10.0, 0.0, -1.0);
drawPalm();

glPushMatrix();
glTranslatef(-1.0, 0.0, 0.0);
drawFingers();
glPopMatrix();

glPopMatrix();

glPopMatrix();
```

Prior to OpenGL 3
Old and new OpenGL?

- Lot has changed from OpenGL 3.
- Everything old still works in compatibility mode.

Prior to OpenGL 3

- glBegin(...)
- glVertex(...)
- glEnd(...)
- glTranslate(...)
- glRotate(...)
- glScale(...)
- glMaterial(...)

OpenGL 3+

- Vertex Array Object (VAO)
- Vertex Buffer Object (VBO)
- Use other Matrix library (eg GLM)
- Send your matrices to shaders
- Vertex Buffer Object

Good tutorials: [http://antongerdelan.net/opengl/](http://antongerdelan.net/opengl/)
Multiple objects in the scene

- Useful to think of the scene as a tree

```
Scene
  ▼
  |  ▼
RightHand | LeftHand
  | ▼
  | ▼
  | ... || MV*A
  ▼   ▼
  ▼   ▼
Fingers | MV*A*B
  ▼   ▼
  ▼   ▼
Finger1 | ... | Finger5
  ▼   ▼   ▼
  | ▼   | ▼
  | ▼   | ▼
MV*A*B*C1 | MV*A*B*C5
  | ▼
  | ▼
MV*A*B*C3
```
Next time...

- Graphics pipeline in more detail
- How to define color for our geometry?
- Vertex and fragment shaders
- What else could be done?

Our geometry defined by 4 vertices
A parallelogram...