Previously...

- We define our geometry (points, lines, triangles)
- We apply transformations (matrices)

\[
\begin{pmatrix}
\cos(45^\circ) & -\sin(45^\circ) \\
\sin(45^\circ) & \cos(45^\circ)
\end{pmatrix}
\begin{array}{c}
\phantom{1}
\end{array}
= \quad \begin{array}{c}
\phantom{1}
\end{array}

When is this true?
Now...

What exactly is here?

This isn't quite true...
Material properties

• We want GPU to take into account a color property when rendering some geometry.

Red cube?

Two red trapezoids?

Flat red polygon?
What is color?

- Spectrum of the light reflected off a surface.
- In 3D we can't get away with only saying that something is red.
- We need to say that somewhere we have a some kind of light source.
Directional light

- Ok, we have a light direction
- A surface
- Viewer

Viewer doesn't see surface point 4?
Directional light

- Reality – our surfaces are diffusely reflective!
Diffuse surface

- So all we need now is the angle between the surface normal and the light's direction.

By the way, the scattered light intensities may not be equal in all directions...
See *glossy reflection*.

- Why this angle?
Diffuse surface
Diffuse surface

• The actual light energy per surface unit depends on the angle.

\[
\frac{1}{\cos(45^\circ)} \approx 1.42 \quad \frac{1}{\cos(80.81^\circ)} \approx 6.26
\]
Diffuse surface & directional light

- Given a surface point and a light source, we can calculate the color of that surface point.
- We use cosine between the surface normal and a vector going towards the light source.
Diffuse surface and directional light

- To find the cosine of the angle, we can use a scalar / **dot product** operation.

\[
v \cdot u = |v| \cdot |u| \cdot \cos(\text{angle}(u, v))
\]

Geometric definition

\[
v \cdot u = v_1 \cdot u_1 + v_2 \cdot u_2 + v_3 \cdot u_3
\]

Algebraic definition

- Because we now have normals, geometric definition simplifies to:

\[
v \cdot u = |v| \cdot |u| \cdot \cos(\alpha) = 1 \cdot 1 \cdot \cos(\alpha) = \cos(\alpha)
\]
Diffuse surface and directional light

- So if we put those two definitions together:

\[ \mathbf{v} \cdot \mathbf{u} = v_1 \cdot u_1 + v_2 \cdot u_2 + v_3 \cdot u_3 = \cos(\alpha) \]

This should be quite easy for the computer to calculate...

- Dot product and the cosine between two vectors is used quite often in CG.
Diffuse surface and directional light

- Dot product of two vectors \( u \) and \( v \) is the same as vector multiplication.

\[
\mathbf{v} \cdot \mathbf{u} = v_1 u_1 + v_2 u_2 + v_3 u_3 = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \mathbf{v}^T \mathbf{u}
\]

- So for our surface point we get:

\[
\text{Intensity} = \text{directionTowardsLight}^T \cdot \text{surfaceNormal}
\]

\[
I = l^T \cdot n
\]

\[
I \in [0, 1]
\]

What is the visual result of that?
Diffuse surface and directional light

- Two things were missing:
  - Intensity of the light source
  - Color of our material
- Also, this will apply to each of 3 RGB channels.

\[
I_R = n^T \cdot l \cdot L_R \cdot M_R \\
I_G = n^T \cdot l \cdot L_G \cdot M_G \\
I_B = n^T \cdot l \cdot L_B \cdot M_B
\]

- Light that light source emits
- Light that material reflects
Diffuse surface and directional light

What color are the apples if red light shines upon them?

What is wrong with this example? (2+ things)
Point light

- Point lights work the same way, but the light source is a point.
Point light

- Sometimes distance attenuation parameters are added.
- In OpenGL:
  \[
  \text{attenuation} = \frac{1}{k_c + k_r d + k_q d^2}
  \]
  
  Usually 1 (why?)

- In THREE.js:

  ```javascript
  PointLight(hex, intensity, distance)
  ```

  *Distance - If non-zero, light will attenuate linearly from maximum intensity at light position down to zero at distance.*

  http://threejs.org/docs/#Reference/Lights/PointLight
Ambient light

- So, now we have 2 lights and a diffuse surface.
- Are we OK?
Ambient light

- World contains much more than 1 cube and a light source.
- Do you know what scene that is?
- Calculating every reflection from every other object is time-consuming.
- What can we do?
Ambient light

- Ambient light source – estimates the light reflected off of other objects in the scene
- Ambient material property – how much object reflects that light (usually same as diffuse)
Lambert material

- So together with diffuse lighting we get:

\[ I_R = L_{AR} \cdot M_{AR} + n^T \cdot l \cdot L_{DR} \cdot M_{DR} \]

\[ I_G = L_{AG} \cdot M_{AG} + n^T \cdot l \cdot L_{DG} \cdot M_{DG} \]

\[ I_B = L_{AB} \cdot M_{AB} + n^T \cdot l \cdot L_{DB} \cdot M_{DB} \]

Red channel

Green channel

Blue channel

Ambient term

Diffuse term

What could go wrong?
Is this it?

- Well, we've already made a very rough approximation of reality with the ambient term.
- But is there anything else that we've forgotten?
Specular highlight

- Depends on the viewer's position.
- We have to get back to the angles.
- At point 4, which viewer direction should produce more specular highlight?
- How to calculate that based on $\beta$?
Specular highlights

- Ok, let's add a specular term based on the actual reflection direction \((r)\) and viewer direction \((v)\).

\[
I_R = L_{A_R} \cdot M_{A_R} + n^T \cdot l \cdot L_{D_R} \cdot M_{D_R} + r^T \cdot v \cdot L_{S_R} \cdot M_{S_R}
\]

\[
I_G = L_{A_G} \cdot M_{A_G} + n^T \cdot l \cdot L_{D_G} \cdot M_{D_G} + r^T \cdot v \cdot L_{S_G} \cdot M_{S_G}
\]

\[
I_B = L_{A_B} \cdot M_{A_B} + n^T \cdot l \cdot L_{D_B} \cdot M_{D_B} + v^T \cdot r \cdot L_{S_B} \cdot M_{S_B}
\]

Light properties are usually same in same channel.

Is there something missing?
Specular highlights

- Let's calculate specular highlight for different angles:

<table>
<thead>
<tr>
<th>$M_s$</th>
<th>$L_s$</th>
<th>$\alpha$</th>
<th>$\sim \cos(\alpha)$</th>
<th>$\sim I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>1</td>
<td>10°</td>
<td>0.98</td>
<td>0.25</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>20°</td>
<td>0.94</td>
<td>0.24</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>30°</td>
<td>0.87</td>
<td>0.22</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>40°</td>
<td>0.77</td>
<td>0.19</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>50°</td>
<td>0.64</td>
<td>0.16</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>60°</td>
<td>0.5</td>
<td>0.12</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>70°</td>
<td>0.34</td>
<td>0.09</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>80°</td>
<td>0.17</td>
<td>0.04</td>
</tr>
<tr>
<td>0.25</td>
<td>1</td>
<td>90°</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

This is actually too little change in the result for such a big change from 10° to 20°.

This is too much for such big angles.

Assume we're dealing with one channel (e.g. red)
Assume the channel values are between [0, 1] (mapped later to [0, 255])
Specular highlights

- How to increase the contrast? Use a power.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\sim \cos^2(\alpha)$</th>
<th>$\sim I$</th>
<th>$\sim \cos^3(\alpha)$</th>
<th>$\sim I$</th>
<th>$\sim \cos^4(\alpha)$</th>
<th>$\sim I$</th>
<th>$\sim \cos^5(\alpha)$</th>
<th>$\sim I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>0.97</td>
<td>0.24</td>
<td>0.96</td>
<td>0.24</td>
<td>0.94</td>
<td>0.23</td>
<td>0.92</td>
<td>0.23</td>
</tr>
<tr>
<td>20°</td>
<td>0.88</td>
<td>0.22</td>
<td>0.83</td>
<td>0.21</td>
<td>0.78</td>
<td>0.20</td>
<td>0.73</td>
<td>0.18</td>
</tr>
<tr>
<td>30°</td>
<td>0.75</td>
<td>0.19</td>
<td>0.65</td>
<td>0.16</td>
<td>0.56</td>
<td>0.14</td>
<td>0.49</td>
<td>0.12</td>
</tr>
<tr>
<td>40°</td>
<td>0.59</td>
<td>0.15</td>
<td>0.45</td>
<td>0.11</td>
<td>0.34</td>
<td>0.09</td>
<td>0.26</td>
<td>0.07</td>
</tr>
<tr>
<td>50°</td>
<td>0.41</td>
<td>0.10</td>
<td>0.27</td>
<td>0.07</td>
<td>0.17</td>
<td>0.04</td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>60°</td>
<td>0.25</td>
<td>0.06</td>
<td>0.13</td>
<td>0.03</td>
<td>0.06</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>70°</td>
<td>0.12</td>
<td>0.04</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>80°</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>90°</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Specular highlights

- Specify some **shininess** value $c$ for the material

\[
I_R = L_{AR} \cdot M_{AR} + n^T \cdot l \cdot L_{DR} \cdot M_{DR} + (r^T \cdot v)^c \cdot L_{SR} \cdot M_{SR}
\]

\[
I_G = L_{AG} \cdot M_{AG} + n^T \cdot l \cdot L_{DG} \cdot M_{DG} + (r^T \cdot v)^c \cdot L_{SG} \cdot M_{SG}
\]

\[
I_B = L_{AB} \cdot M_{AB} + n^T \cdot l \cdot L_{DB} \cdot M_{DB} + (r^T \cdot v)^c \cdot L_{SB} \cdot M_{SB}
\]

- Ambient approx.
- Lambertian reflectance
- Phong's shininess term

Something still missing?

Phong lighting model
Specular highlights

$c=0$

$c=30$

$c=90$

$c=300$
Blinn-Phong model

- Sometimes Phong's term is replaced with Blinn-Phong's term.
- Instead of viewer direction and reflected light's direction, we use the surface normal and a half angle between the light source and the viewer.
Blinn-Phong model

- There are some differences
- These are not the only two possibilities

DEMO 2: http://cgdemos.tume-maailm.pri.ee/
THREE.JS videos: https://www.udacity.com/course/viewer#!/c-cs291/l-124106593/m-157996647
Standard Graphics Pipeline

**Vertex shader**
- **Vertex transformations**
  - Perspective division, Viewport transformation
  - \( P \cdot V \cdot M \cdot v \)
- **Culling & Clipping**
  - Culling – remember the face directions?
  - Clipping – some parts are out of view

**Fragment shader**
- **Rasterization**
- **Fragment shading**
- **Visibility tests & Blending**
Vertex shader

• Code you send to GPU to operate on each vertex.

```glsl
#version 120

varying vec3 normal;
varying vec4 vertex_pos;

void main(void) {
    gl_Position = ftransform();
    vertex_color_if_lighting_disabled = gl_Color;

    normal = gl_NormalMatrix * gl_Normal;
    vertex_pos = gl_ModelViewMatrix * gl_Vertex;

    // Send the normal and vertex positions down to the fragment shader
    normal = gl_NormalMatrix * gl_Normal;
    vertex_pos = gl_ModelViewMatrix * gl_Vertex;
}
```

Corresponds to OpenGL 2.1
GLSL 1.20.8

varying type is interpolated

This shader needs to set gl_Position

ftransform() is gl_ProjectionMatrix * gl_ModelViewMatrix * gl_Vertex;
Fragment shader

- Code you send to GPU to operate on each fragment (pixel)

```glsl
#version 120

varying vec3 normal;
varying vec4 vertex_pos;
uniform vec3 viewer_pos;

vec4 frag_color = vec4(0, 0, 0, 0);
void addLight(int lightIndex);

void main(void) {
    frag_color = gl_LightModel.ambient * gl_FrontMaterial.ambient;
    addLight(0);
    gl_FragColor = frag_color;
}
```

Uniform variables you send to the shader

OpenGL will give lights info (like the ambient term).

Must set the gl_FragColor
void addLight(int lightIndex) {
    float attenuation = 1.0; // no attenuation
    vec3 light_dir = normalize(gl_LightSource[lightIndex].position.xyz);

    vec3 light_reflection = 2 * normal * dot(light_dir, normal) - light_dir;
    frag_color += attenuation *
        gl_FrontMaterial.diffuse *
        gl_LightSource[lightIndex].diffuse *
        clamp(dot(normal, light_dir), 0, 1);
    frag_color += attenuation *
        gl_FrontMaterial.specular *
        gl_LightSource[lightIndex].specular *
        pow(clamp(dot(viewer, light_reflection), 0, 1), gl_FrontMaterial.shininess);
}

Lights in OpenGL are indexed

What lighting model is this?

Why clamp? What's clamp??
#version 330
layout(location = 0)in vec4 position;

uniform mat4 projection;
uniform mat4 view;
uniform mat4 model;

void main() {
    gl_Position = projection * view * model * position;
}

#version 330
out vec4 outputColor;

void main() {
    outputColor = vec4(1.0f, 1.0f, 1.0f, 1.0f);
}

Extensive tutorial: http://arcsynthesis.org/gltut/
Raytracing

- Cast a ray and find the closest object it hits.
- Recurse from the hitpoint.
- Where to cast a ray from and where to?
  - Camera, pass through some pixel in screen
  - From hit point to reflection direction
  - From hit point to refraction direction
  - From a front of a spaceship to forwards direction
Raytracing

- Realistic reflections
- Realistic lighting
- Realistic shadows
Raytracing

• How to find a hit point?
• I assume at this point I don't have much time left, so... I recommend to watch this: https://www.youtube.com/watch?v=zQlvrR_6layM
• Basic idea is to find an intersection of the ray and a plane defined by a triangle.
Raytracing

- Usually quite resource consuming
- Can be optimized with:
  - Bounding Volume Hierarchies
  - Space Partitioning

Example of one space partitioning structure... there are many.
Conclusion

- Computer graphics can be used to create a illusion of reality
- First approximation is of the shape – geometry
- GPU wants those triangles
- Vertices and transformation matrices
Conclusion

- Many ways to approximate lighting (Lambert, Phong, Blinn), reflections, refractions, shadows...
- Ambient, diffuse, specular terms

\[ I = L_A \cdot M_A + n^T \cdot l \cdot L_D \cdot M_D + (r^T \cdot v)^c \cdot L_S \cdot M_S \]

Direction towards light, surface normal, reflection direction, direction towards viewer
Thanks for listening!