# Augmented Reality and Point Set Matching 

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## What is Augmented Reality (AR)?

- Mixed interactive experience - real-world environment with computer-generated information
- Visual
- Auditory
- Optionally, haptic, neural, smell
- System that fulfils 3 conditions:
- A combination of real and virtual worlds
- Real-time interaction
- Accurate 3D registration of virtual and real objects


## Current Use Cases?

- Not used often in commercial products
- Furniture
- Visual aid
- "Augments" the environment


## Devices

- Camera - to perceive the world and make decisions from it
- A computer with a camera
- Smart devices (phones, tablets)
- Head-Mounted Display


## Virtual Reality or Augmented Reality?

- VR - The perception of reality is all based on virtual information



## Virtual Reality or Augmented Reality?

- AR - Part of the environment is "real", with virtual objects on top of it


Augmented Reality History

## 1968

- Note - VR and AR didn't exist at this point, only Artificial Reality


## 1968

- The Sword of Damocles
- Created by Ivan Sutherland
- 2 CRT screens for either eye
- Screens display 2D objects at different angles
- Head position sensors
- Wireframe objects



## 1969

- Glowflow
- Myron Krueger
- Wanted Artificial Reality that didn’t need goggles or gloves to interact
- Dark room filled with phosphorescent pigments
- Floor sensors reacted to movement, synthesizer sounds, origin, lighting

- Issues with trigger mechanisms


## 1970

- Metaplay
- 2 Rooms connected through a video feed
- "Artist" sees others through camera, can interact with a drawing tablet
- Participants saw themselves project
 artist can write and draw on


## 1971

- Psychic Space
- Advancement of Glowflow
- Improved sensor detection
- Invisible maze, user location displayed on screen with "wall" locations


## 1974

- Videoplace
- 2 rooms where image of one side projected to other. Both participants see the same image
- More interactivity
- Playing with a virtual ball

- Changing their displayed image, resizing, rotating
- Typing text
- Computer Vision



## 1990

- The term "Augmented Reality" was used for the first time
- Thomas Caudell (David Mizell)
- Developed an industrial AR head mounted display
- Displays computer-generated diagrams of the manufacturing process
- Branched off as industrial AR (supporting industrial work)


## 1992

- Virtual Fixtures
- Louis Rosenberg
- First fully functional AR system
- Displays and overlays virtual sensor information
-3D graphics slow at that time, uses 2 physical robots instead
- Binocular magnifiers aligned user view with robot arms for better immersion



## 1994-1998

- Various entertainment use cases
- Sportsvision displaying a yellow line in American Football games
- Individually displayed and updated for every camera showing games
- Samples field / players for line occlusion effects



## 1999

- NASA hybrid synthetic vision system for spacecraft


## 2000

- Open-source software library ARToolKit developed.
- Video tracking to overlay virtual graphics on top of it.
- Still being updated today


## 2001-2013

- 2003 - NFL usage of Skycam for virtual markers
- 2009 - Esquire Magazine with scannable barcodes to display AR content
- 2013 - Volkswagen using AR as car manuals (MARTA)


## 2014,2016

- Google Glass, Microsoft Hololens
- More immersive alternatives to smartphones
- Google Glass had a camera, touchpad, voice commands.
- Hololens have an accelerometer, gyroscope, magnetometer, depth-camera, multiple microphones, light sensor.


The Future?


## Feature Detection

## Feature Detection

- A subcategory of computer vision and image processing
- Methods to compute image information
- "Feature" means "interesting" part of an image
- 4 general types
- Edges
- Corners (Interest points)
- Blobs (Regions of interest points)
- Ridges


## Edge Detection

- Detect parts where brightness changes sharply
- Good in image processing, not AR.
- Marker-based solutions could still use this (searching for specific image)



## Corners

- An interest point where there is an intersection of 2 edges.
- Ends of a parabola
- Markerless AR



## Blobs

- General analysis of image
- Find regions that differ in properties
- Brightness
- Color
- Smooth areas
- Markerless AR



## Ridges

- For elongated objects
- 1D curves
- Harder to compute
- Road detection on aerial images



## Object Placement

- Once the system understands the environment, it needs a way to place objects:
- Marker-based
- Markerless
- Location based


## Marker-Based

- Virtual object placed on a "marker"
- Detectable image
- Can only visualize one object per image


Marker-Based


## Markerless

- Uses feature points to detect surfaces
- Generates planes with feature points (plane detection)
- Objects placeable on planes.
- Only supports horizontal/vertical surfaces
- Doesn't work with flat colored surfaces



## Location based

- Uses real-world coordinates to estimate where to place objects
- Latitude, longitude, altitude
- Better for outdoors environments
- More advanced features currently only on iOS


## What about point cloud based?

- Master thesis study - utilize AR on a lower level
- Detecting and saving feature points as point clouds
- Comparing point clouds and matching them
- Saved point cloud can have special object locations added to visualize once point clouds matched.


## Problem 1 - Persistent Data

- Feature points are deleted when no longer seen by a device
- A separate container to store all visible points


## Problem 2 - Excessive Data

- Too many points are saved, causes slowdown
- Check nearby points to see if a point should be added?
- Would require traversing the entire collection of points for each point $O(n)$
- Octrees
- 3D cube dividing to a minimum size
- Only compare points within divided cube
- Finding time now $O(\log n)$


## Problem 3 - Saving Point Clouds

- A way to save and load point clouds in some form of data
- Each point is a 3D position
- Write the coordinates as a binary stream


## Problem 4 - Comparing Point Clouds

- Many potential issues
- Individual points, not planes - lots of comparisons between points
- Tens of thousands of points per PC
- Unique number of points per PC
- Many points that can't be directly matched with each other
- Inaccurately placed points (outliers)
- Coordinate systems of points clouds not matching


## Point Set Registration

- Finding a spatial transformation that aligns two point clouds
- Scaling
- Rotation
- Translation
- 2 finite point sets in a finite-dimensional real vector space


## Point Set Registration

- Finding a spatial transformation that aligns two point clouds
- Scaling
- Rotation
- Translation
- 2 finite point sets in a finite-dimensional real vector space
- Transformation types:
- Rigid Registration -2 separate point clouds matched without the distance between 2 points of a point cloud changing. Just translation and rotation of the point clouds
- Non-Rigid Registration - allows non-linear transformation, scaling included


## Point Set Registration

- One of the issues was point having deviations (outliers, different point locations)
- Non-rigid should be the solution
- Algorithms that cover this area:
- SG4PCS
- 2PNS
- ACPD


## Before That

- RANSAC - Random Sample Consensus.
- Picks 3 random points from either point cloud
- Computations to count points from one point cloud that are close to points in other.
- If point count large enough, accepted as answer.
- Otherwise it repeats.


## 4-Point Congruent Sets (4PCS)

- "Fast", robust alignment scheme for 3D point sets.
- Resilient to noise and outliers, even with small overlap


## 4-Point Congruent Sets (4PCS)

-1. Uses coplanar sets of 4 point rather than minimum of 3 , to apply a technique to match pairs of affine invariant ratios in 3D

- Coplanar - Same plane

- Affine - Preservation of lines and parallelism, but not distance
- Invariant - Property that remains unchanged after transformations

$\triangle A B C \cong \triangle D E F$


## 4-Point Congruent Sets (4PCS)

- 2. Select a base of 4 coplanar points in PC $P$
- Find all 4-point sets in target PC $Q$, that are approximately congruent with base points.
- Done in $O\left(n^{2}+k\right)$ time
- $n$ - Number of points in $Q$
- $k$ - Number of 4-point sets.


## 4-Point Congruent Sets (4PCS)

- 3. For each 4-point set from $Q$, compute aligning transformation $T$, retain best transformation based on Largest Common Pointset score.
- Repeat in RANSAC scheme until good solution found or maximum iterations reached.
- LCP problem
- Given 2 point sets $P$ and $Q$, under $\delta$-congruence, Find largest countable subset of $P$ called $P^{\prime}$ where distance between $T\left(P^{\prime}\right)$ and $Q$ is less than $\delta . T$ being a rigid transform.


## 4-Point Congruent Sets (4PCS)

- 4. First step of each RANSAC iteration - pick a random base of 4 coplanar points.
- Picks first 3 randomly to create a wide triangle.
- $4^{\text {th }}$ selected is close to the planar of the other 3.
- Testing all $S$ point in $P$, picking the one that fits best

- Complexity $O(S)$


## 4-Point Congruent Sets (4PCS)

- 5. $\mathcal{B}=\{A, B, C, D\}$, where $E$ is intersection of $A B$ and $C D$
- $r_{1}=\frac{\|A-E\|}{\|A-B\| \mid}, r_{2}=\frac{\|C-E\|}{\|C-D\|}$
- $d_{1}=||A-B||, d_{2}=||C-D||$
- Ratios $r_{1}$ and $r_{2}$ remain invariant under affine transformation, and therefore under rigid motion
- Distances preserved with rigid transformations - these 4 invariants used for searching congruent 4-point sets in $Q$



## 4-Point Congruent Sets (4PCS)

- $r_{1}=\frac{\|A-E\|}{\|A-B\|}, r_{2}=\frac{\|C-E\|}{\|C-D\|}$
- $d_{1}=\|A-B\|, d_{2}=\|C-D\|$
-6. Extract all pairs of points and distance $d_{1}$ or $d_{2}$ from $Q$
- $O\left(N^{2}\right)$ time
- $N$ - amount of points in $Q$
- For each extracted pair $\left(Q_{1}, Q_{2}\right) \in Q$ with distances $d_{1}$ or $d_{2}$, compute intermediate points $E_{1}, E_{2}$



## 4-Point Congruent Sets (4PCS)

- For each extracted pair $\left(Q_{1}, Q_{2}\right) \in Q$ with distances $d_{1}$ or $d_{2}$, compute intermediate points $E_{1}, E_{2}$
-7. 2 pairs whose $E_{1}, E_{2}$ are coincident form a 4 -point base related with $\mathcal{B}$ by an affine transformation
- Intermediate points from pairs at $d_{1}$ used to build an approximate range tree structure
- $O(M \log M)$, where $M$ is number of pairs
- Query time $O(\log M+K)$, where $K$ is number of points needed to get


## 4-Point Congruent Sets (4PCS)

- Intermediate points from pairs at $d_{1}$ used to build an approximate range tree structure
- $O(M \log M)$, where $M$ is number of pairs
- Query time $O(\log M+K)$, where $K$ is number of points needed to get
- 8. Intermediate points from pairs at $d_{2}$ used to query the tree
- Result is K 4-point sets from pairs
- $O(K)$ time to remove non-rigid sets


## Super 4-Point Congruent Sets (S4PCS)

- Improves certain search stages to decrease complexity from quadratic to linear time.
- Supposedly works with about $25 \%$ overlap with $20 \%$ outlier margin.
- Complexity decreased to $O(N+M+K)$ by solving 2 bottlenecks
- Pair Extraction (2) - $O\left(N^{2}+K\right)$
- Verification (8) - $O(K)$


## Super 4-Point Congruent Sets (S4PCS)

- First bottleneck
- 2. Select a base of 4 coplanar points in PC $P$
- Find all 4-point sets in target PC $Q$, that are approximately congruent with base points.
- Done in $O\left(n^{2}+k\right)$ time
- $n$ - Number of points in $Q$
- $k$ - Number of 4-point sets.


## Super 4-Point Congruent Sets (S4PCS)

- 2. Select a base of 4 coplanar points in PC $P$
- Find all 4-point sets in target PC $Q$, that are approximately congruent with base points.
- Now find points close to spheres centered in $Q_{i} \in Q$ with radius $d_{1} \pm \epsilon$ and $d_{2} \pm \epsilon$, where $\epsilon$ is noise tolerance
- Pair extraction reduced to $O(n)$



## Super 4-Point Congruent Sets (S4PCS)

- Second bottleneck
- Intermediate points from pairs at $d_{1}$ used to build an approximate range tree structure
- $O(M \log M)$, where $M$ is number of pairs
- Query time $O(\log M+K)$, where $K$ is number of points needed to get
- 8. Intermediate points from pairs at $d_{2}$ used to query the tree
- Result is K 4-point sets from pairs
- $O(K)$ time to remove non-rigid sets


## Super 4-Point Congruent Sets (S4PCS)

- $O(K)$ time to remove non-rigid sets
- Now extract only congruent 4-point bases that are rigid-invariant, so verification not needed.
- 4-point set congruent to base from $P$ if it's composed of pairs with correct length ( $d_{1}, d_{2}$ ) and angle $\phi$ between them similar to angle formed by the 2 pairs in the base


## Super 4-Point Congruent Sets (S4PCS)

- Represent each point by intermediate point $E$ and orientation.
- $d_{1}$ pairs hashed by this position and orientation, mapped to a spherical map.
- In query stage (7), get cells in a regular grid using $E$.
- Query sphere map using a $d_{2}$ pair direction, find all pairs with angle $\phi$ in regards to query direction. A cone of aperture $2 \phi$ is intersected around the query direction.
- Complexity $O(M+K)$


## Super 4-Point Congruent Sets (S4PCS)



## Generalized 4-Point Congruent Sets (G4PCS)

- Alternate advancement of 4PCS
- Different definition of the 4-point base
- $\mathrm{X}=\{A, B, C, D\}$, where AB does NOT always lie on same plane as $C D$
- $r_{1}=\frac{\|a-x\|}{\|a-b\|}, r_{2}=\frac{\|c-y\|}{\|c-d\|}$
- $d_{3}=||x-y||$



## Generalized 4-Point Congruent Sets (G4PCS)

- Predefining values of $d_{1}, d_{2}, d_{3}$ to sample only bases that satisfy them.
- Any wide base now sampled from $P$, then $d_{i}$
- Bases storable in a 2D hash table based on ratios $r_{1}, r_{2}$
- Should lessen the amount of bases found



## Super Generalized 4PCS (SG4PCS)

## - Combination of S4PCS and G4PCS

```
Algorithm 1 The Super Generalized 4PCS Algorithm
Input: Target and source point sets, P and Q
Output: Best transformation according to LCP, T}\mp@subsup{T}{\mathrm{ best}}{
    d}=\mp@subsup{d}{2}{}=\mathrm{ fractional_overlap }\times\mathrm{ model_diameter
    Extract d}\mp@subsup{d}{1}{},\mp@subsup{d}{2}{}\mathrm{ pairs from }
    Extract }\mp@subsup{d}{1}{},\mp@subsup{d}{2}{}\mathrm{ pairs from }
    Initialize a 4D hash table H to store intersecting pairs
    Compute all valid 3D intersections in Q and store in H
    L= number of RANSAC iterations
    Tbest}=\mathbf{0
    for l=0 to L do
        B= random base from P
        r}\mp@subsup{r}{1B}{},\mp@subsup{r}{2B}{},\mp@subsup{d}{3B}{},\mp@subsup{\alpha}{B}{}\mathrm{ are invariants of }
        C=ExtractCongruent( }\mp@subsup{r}{1B}{},\mp@subsup{r}{2B}{},\mp@subsup{d}{3B}{},\mp@subsup{\alpha}{B}{}
        TB}=\mathrm{ Transformation with highest LCP from C
        if LCP(T (Test })<\operatorname{LCP}(\mp@subsup{T}{B}{})\mathrm{ then
            Tbest }=\mp@subsup{T}{B}{
        end if
    end for
```


## 2 Point Normal Sets (2PNS)

- Alternative to S4PCS using a different approach to 3D registration
- Using normals instead
- Rigid transformation $T$ computable from 2 points plus normal of one point
- Reduces needed comparisons


## 2 Point Normal Sets (2PNS)

- 1. Computing point normals
- PC surface normal estimation, PlanePCA?
- 2 solutions for each normal vector
- Fails when normals not estimated correctly
- Sparse PC
- Mostly sharp edges and corners


## 2 Point Normal Sets (2PNS)

- 2. 2PNS search to obtain existing matches
- Take 2 points and normals from source PC $P$
- Extracts pairs
- $d=||A-B||$
- angle $\theta=\angle\left(n_{A}, n_{B}\right)$
- Verify 3 additional angles to prevent non-rigid
 solutions
- Angles preserved under rigid transformation


## 2 Point Normal Sets (2PNS)

-3. Estimation of R

- Let ( $A^{\prime}, B^{\prime}$ ) with normals $n_{A}, n_{B}$, be pair of points in PC $Q$, congruent with pair in $P$
- Estimate rigid transformation and compute their rotation $R=\mathrm{R}_{\alpha} \cdot R_{\beta}$
- $R_{a}$ aligns vectors

- $R_{\beta}$ aligns normal vectors


## 2 Point Normal Sets (2PNS)

- Estimate rigid transformation and compute their rotation $R=\mathrm{R}_{\alpha} \cdot R_{\beta}$
- $R_{a}$ aligns vectors
- $R_{\beta}$ aligns normal vectors
- $R_{\alpha}$ simple rotation to align 2 vectors
- $v_{1}=B-A, v_{2}=B^{\prime}-A^{\prime}$
- $\omega_{\alpha}=v_{1} \times v_{2}$
- $\alpha=\cos ^{-1}\left(v_{1} \cdot v_{2}\right)$
- $R_{\beta}$ found by rotating angle $\beta$ around axis $v_{2}$

- $n_{P}^{*}=R_{\alpha}$
- $n_{P}, P=A, B$
- $\beta^{\prime}=\pi-\beta$
- Translation estimated from $R$


## Coherent Point Drift (CPD)

- Probabilistic approach to align point sets.
- Consider problem as probability density estimation problem, fit Gaussian Mixture Model centroids by maximizing likelihood.
- Let $X_{\{M \times d\}}=\left(x_{1}, x_{2}, \ldots, x_{M}\right)^{T}$ be the template PC

and $Y_{\{N \times d\}}=\left(y_{1}, y_{2}, \ldots, y_{N}\right)^{T}$ the target PC
- $d$ - PC dimension (3)
- M and N - Amount of points in X and Y .
 estimation problem, fit Gaussian Mixture Model


## Coherent Point Drift (CPD)

- $X_{\{M \times d\}}=\left(x_{1}, x_{2}, \ldots, x_{M}\right)^{T}$ template PC
- $Y_{\{N \times d\}}=\left(y_{1}, y_{2}, \ldots, y_{N}\right)^{T}$ target PC
- Uses weighted GMM probability density function. Noise or outliers accounted as
- $p(y)=\omega \frac{1}{N}+(1-\omega) \sum_{(m=1)}^{M} \frac{1}{M} p(y \mid m)$

- $p(y \mid m)=\left(\frac{1}{\left(2 \pi \sigma^{2}\right)^{\frac{d}{2}}}\right) \exp \left(-\left(\frac{\left\|y-x_{m}\right\|^{2}}{2 \sigma^{2}}\right)\right)$
- $\omega$ - Weight of uniform distance between 0 to 1.
- $\sigma$-Standard deviation


## Coherent Point Drift (CPD)

- Next step - using expectation-maximization scheme (EM) to find final 3D rigid transformation.
- E-step (posterior probability of GMM)
$\cdot P^{(i)}\left(m \mid y_{n}\right)=\frac{\exp \left(-\left.\frac{1}{2}| | \frac{y_{n}-T\left(x_{n}, \phi^{(i)}\right)}{\sigma^{(i)}}\right|^{2}\right)}{\sum_{k=1}^{M} \exp \left(-\frac{1}{2}| | \frac{y_{n}-T\left(x_{m}, \phi^{(i)}\right.}{\sigma^{(i)}}| |^{2}\right)+\left(2 \pi\left(\sigma^{(i)}\right)^{2}\right)^{\frac{d}{2} \omega M}} 1-\omega N$.
- Calculate correspondence probability matrix
- $P=\left[P\left(m \mid y_{n}\right)\right]_{M \times N}$


## Coherent Point Drift (CPD)

- M-step, new parameter set calculated by maximizing auxiliary function $Q\left(\Theta, \Theta^{(i)}\right)$, upper bound for log-likelihood function
- $L(\Theta)=\log \left(\Pi_{n=1}^{N}\left(p\left(y_{n}\right)\right)=\sum_{n=1}^{N} \log \sum_{m=1}^{M+1} P(m) p\left(y_{n} \mid m\right)\right.$


## Accelerated Coherent Point Drift (ACPD)

- CPD suffers from high computational complexity / convergence to local minima.
- ACPD offers 2 methods to speed up performance


## Accelerated Coherent Point Drift (ACPD)

-1. Speed up Expectation-Maximization

- gSQUAREM
- 2. Calculating probability matrix $P$ more efficiently
- DT-IFGT
- Reduces $O(M \cdot N)$ to $O(M+N)$


