GeoGebra

Computer Graphics Seminar

MTAT.03.296

Spring 2014

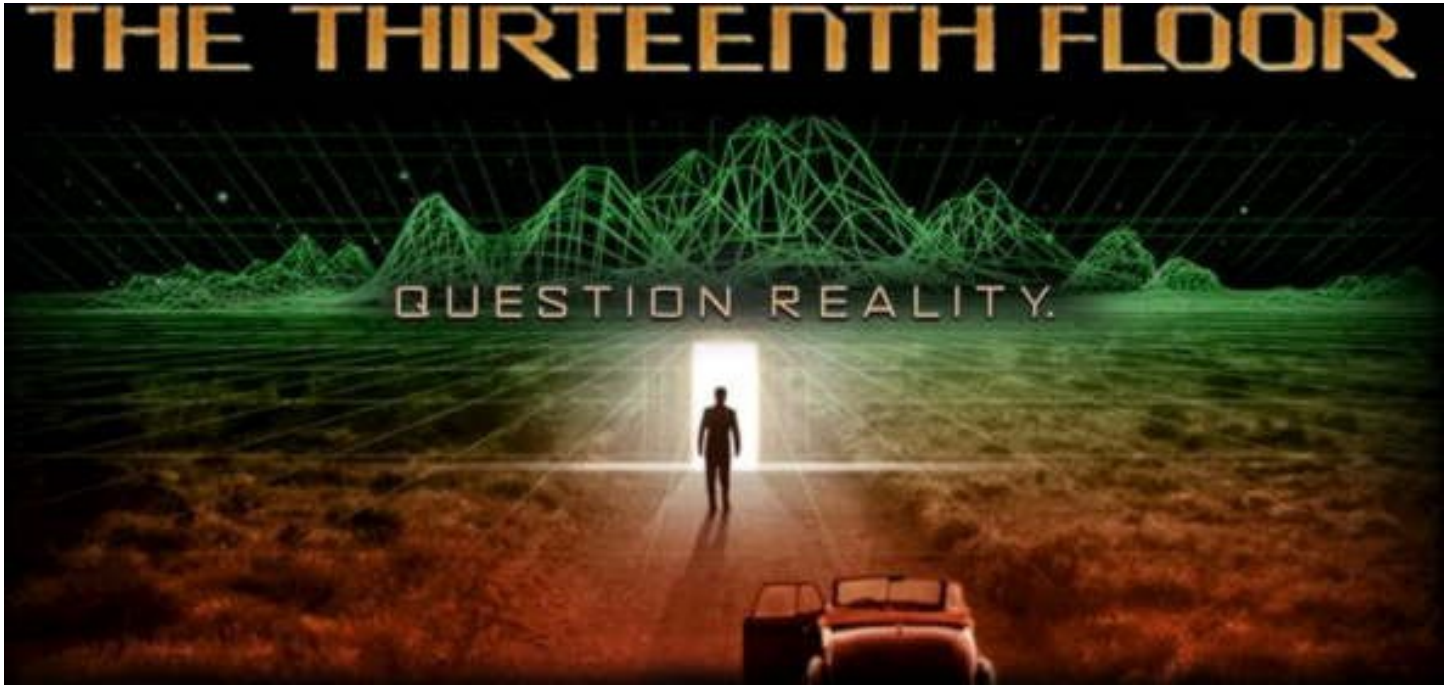
Raimond Tunnel

TARTU ÜLIKOOL · UNIVERSITAS TARTUENSIS · 1632

IT Akadeemia
toetab Skype™

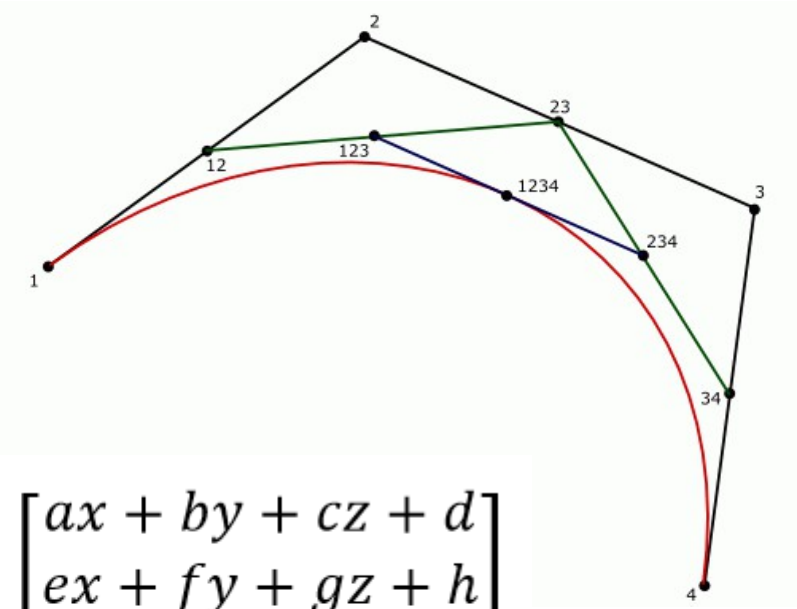
Computer Graphics

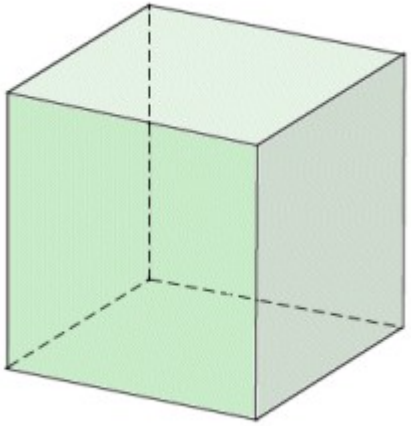
- Graphical illusion via the computer
- Displaying something meaningful (inc art)



Math

- Computers are good at... computing.
- To do computer graphics, we need math for the computer to compute.
- Geometry, algebra, calculus.





$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + cz + d \\ ex + fy + gz + h \\ ix + jy + kz + l \\ 1 \end{bmatrix}$$

Point

- Simplest geometry primitive
- In homogeneous coordinates:

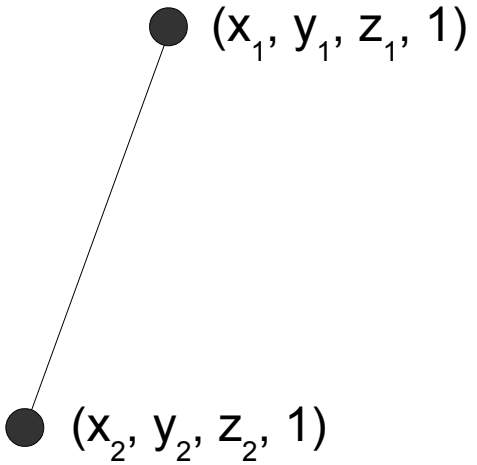
(x, y, z, 1)

$$(x, y, z, w),\ w \neq 0$$

- Represents a point (x/w, y/w, z/w)
- Usually you can put w = 1
- Actual division will be done by GPU later

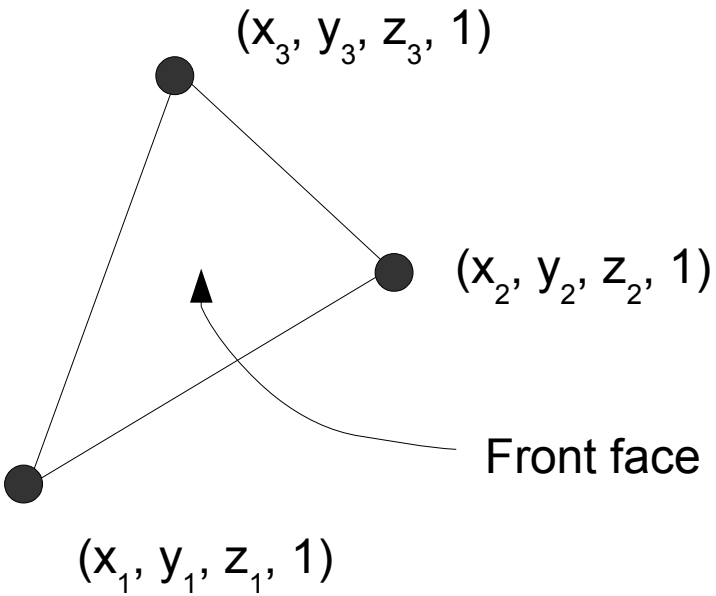
Line (segment)

- Consists of:
  - 2 endpoints
  - *Infinite* number of points between
- Defined by the endpoints
- Interpolated in GPU



Triangle

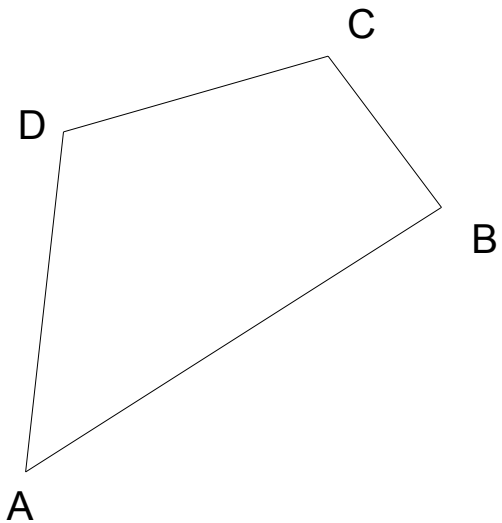
- Consists of:
  - 3 points called vertices
  - 3 lines called edges
  - 1 face
- Defined by 3 vertices
- Lines and face will be interpolated in GPU
- Counter-clockwise order defines front face

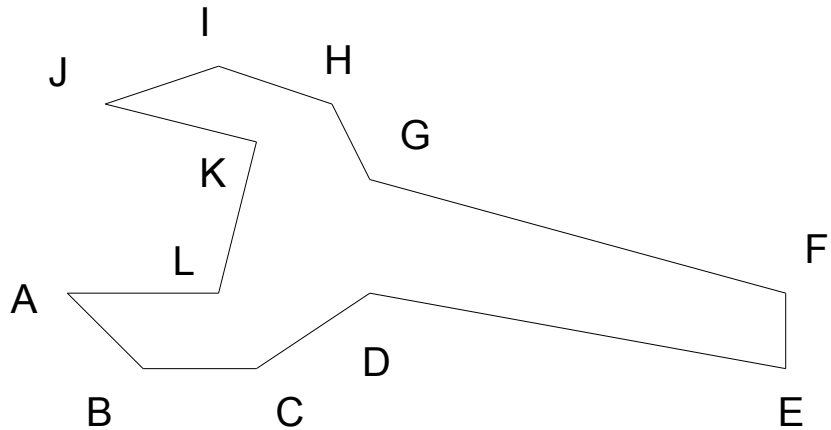


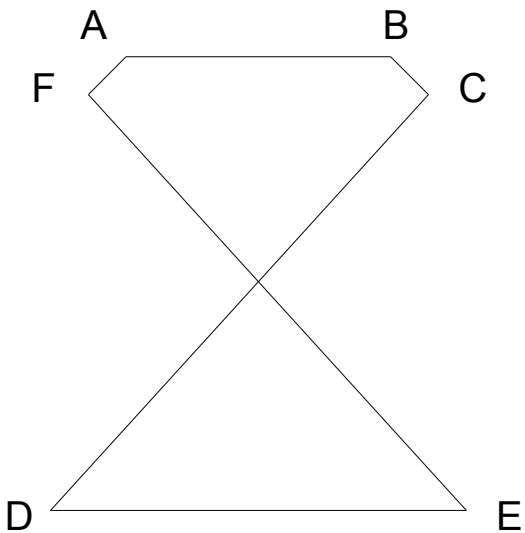
Why triangles?

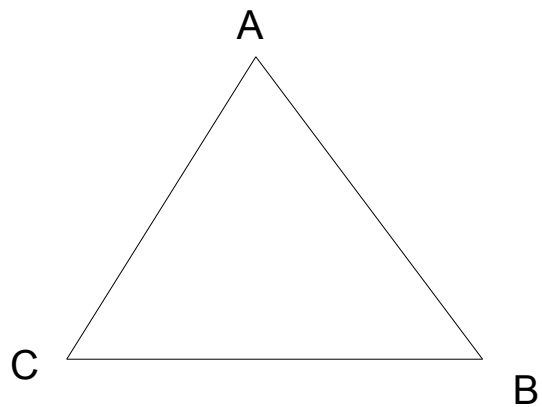
- They are in many ways the simplest polygons
  - 3 different points always form a plane
  - Easy to rasterize (fill the face with points)
  - Every other polygon can be converted to triangles
- OpenGL used to support other polygons too
  - Must have been:
    - Simple – No edges intersect each other
    - Convex – All points between any two points are inner points

Examples of polygons









OpenGL < 3.1 primitives

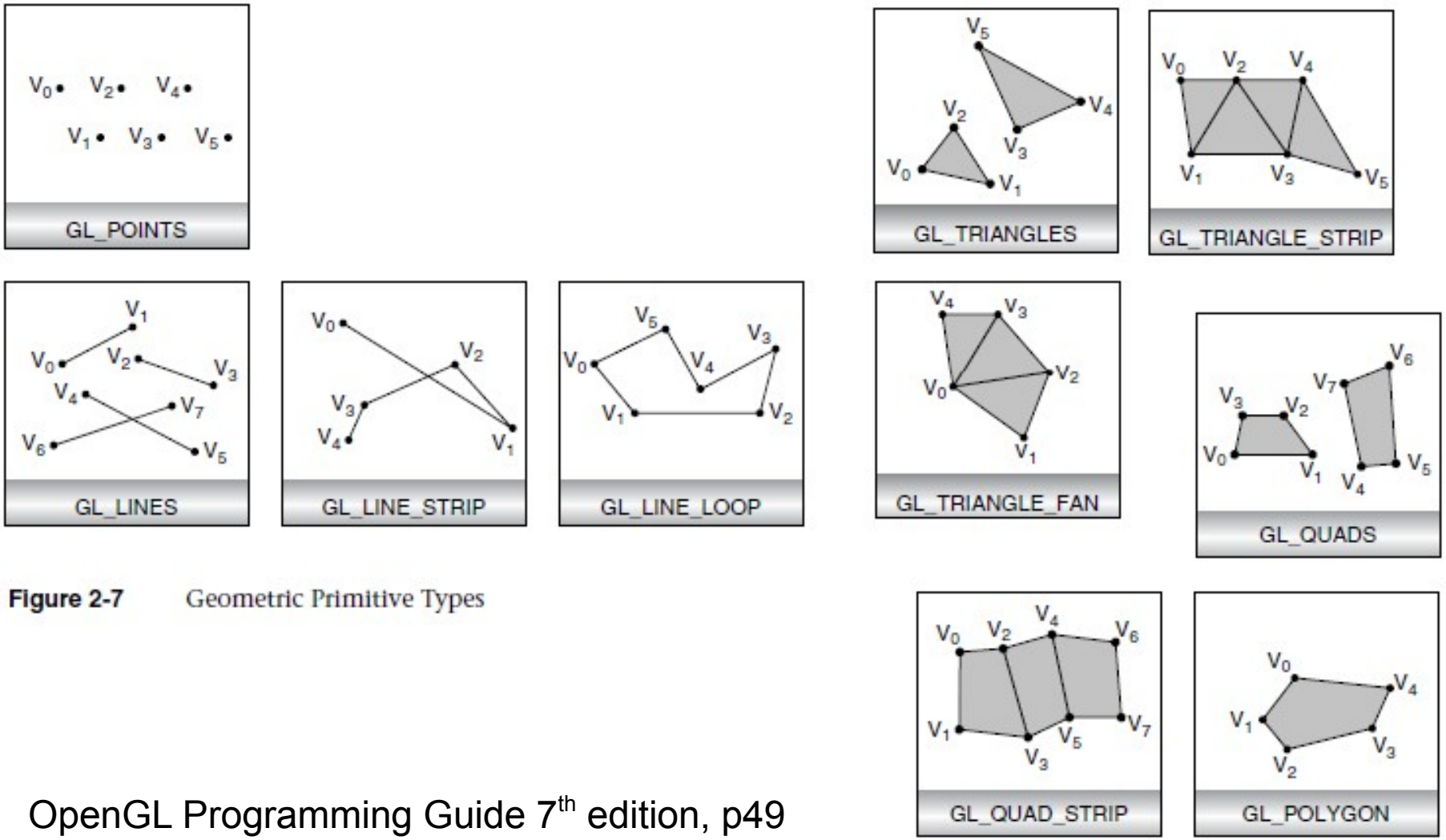


Figure 2-7 Geometric Primitive Types

OpenGL Programming Guide 7th edition, p49

After OpenGL 3.1

Table 3.1 OpenGL Primitive Mode Tokens

Primitive Type	OpenGL Token
Points	GL_POINTS
Lines	GL_LINES
Line Strips	GL_LINE_STRIP
Line Loops	GL_LINE_LOOP
Independent Triangles	GL_TRIANGLES
Triangle Strips	GL_TRIANGLE_STRIP
Triangle Fans	GL_TRIANGLE_FAN

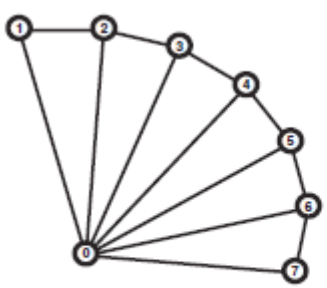


Figure 3.2 Vertex layout for a triangle fan

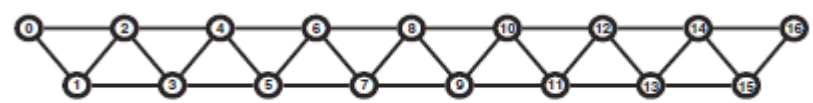


Figure 3.1 Vertex layout for a triangle strip

OpenGL Programming Guide 8th edition, p89-90

In the beginning there were points

- Now that we can define our geometric objects, what next?
- We want to move our objects!

Luckily GPU will do this work for us.
If we tell it to...

Transformations

- It turns out that homogeneous coordinates allows us to easily do:
  - Linear transformations
    - Scaling, reflection
    - Rotation
    - Shearing
  - Affine transformations
    - Translation (shifting)
  - Projection transformations
    - Perspective
    - Orthographic

Actually these we could do without homogeneous coordinates...

This too...

Transformations

- Every transformation is a function
- As you remember from Algebra, linear functions can be represented as matrices

$$f(v)=(2\mathrm{x}, y, z)=\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$v \in R^3$$

$$v=(x, y, z)$$

You should transpose the result later

Transformations

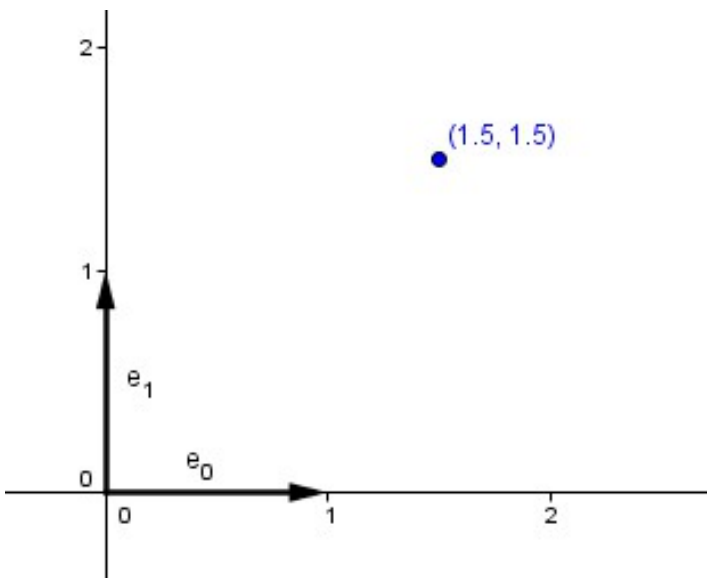
- GPU-s are built for doing transformations with matrices.
- Remember, computers are made for computing.
- Let's look at some linear transformations...

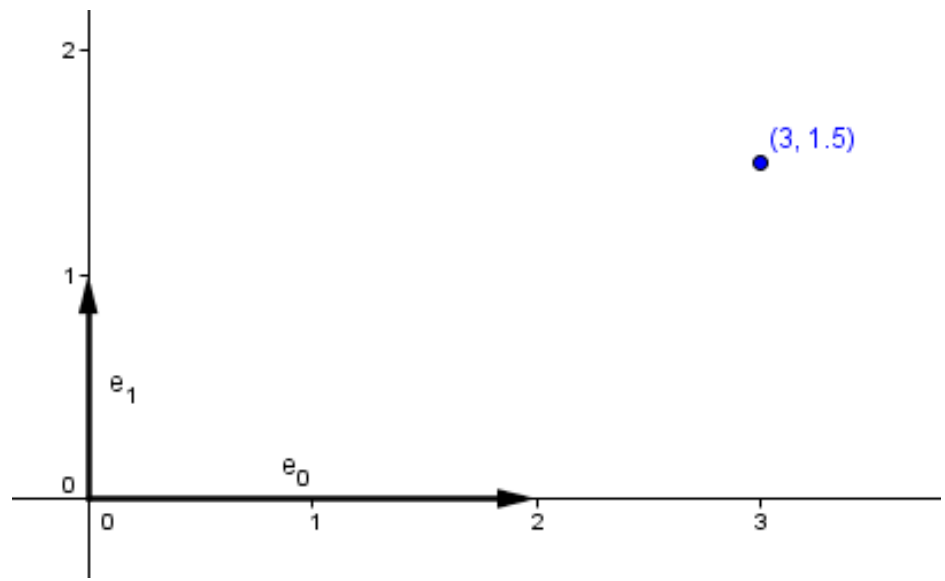
$$f(a_1 x_1 + \ldots + a_n x_n) = a_1 f(x_1) + \ldots + a_n f(x_n)$$

We don't use homogeneous coordinates at the moment, don't worry, they'll be back...

Scaling

- Redefines the basis vectors as some multiple of the previous basis vectors.





$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.5 \end{pmatrix}$$

Scaling

- In general we could scale each axis

$$\begin{pmatrix} a_x & 0 & 0 \\ 0 & a_y & 0 \\ 0 & 0 & a_z \end{pmatrix}$$

a_x – x-axis scale factor

a_y – y-axis scale factor

a_{x} – z-axis scale factor

- If some factor is negative, this matrix will reflect the points from that axis. Thus we get reflection.

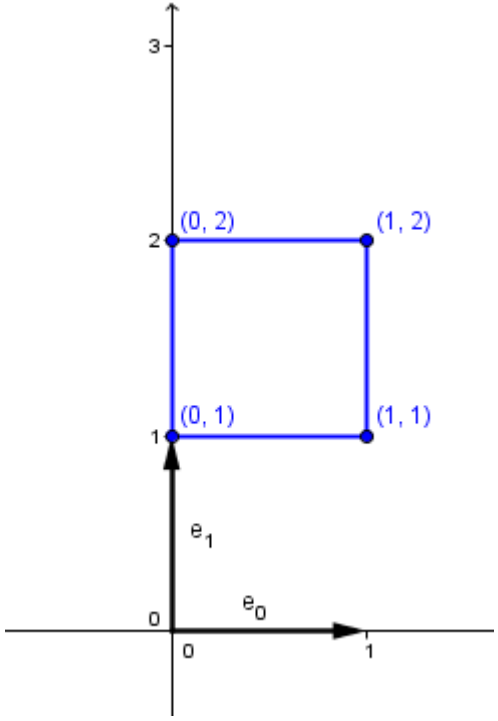
Shearing

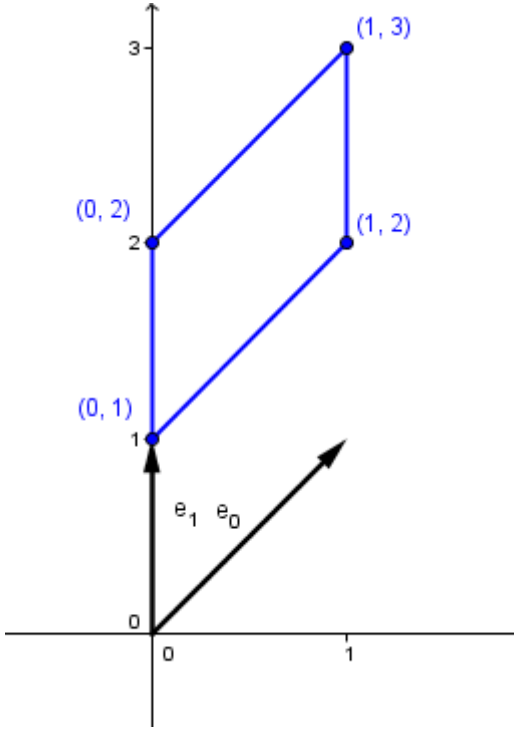
- Not much used by itself, but remember it for translations later.
- Rotates only one axis.
- Squares become parallelograms.

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

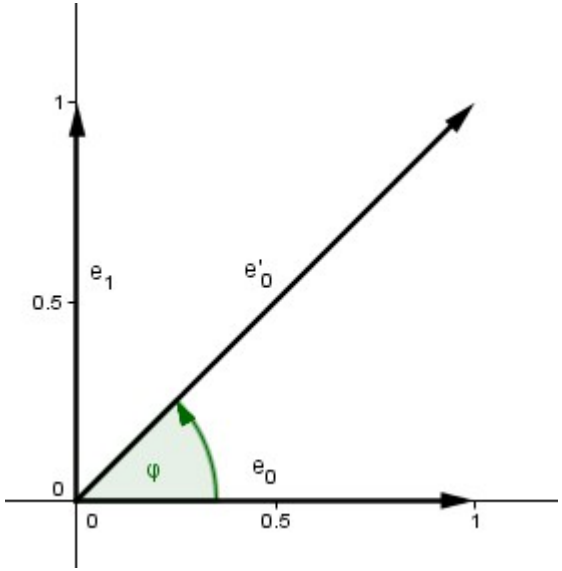




Shearing

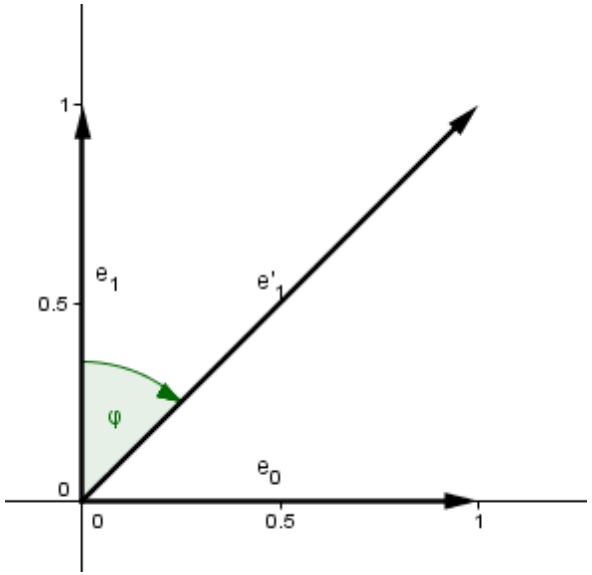
- Shear-x, we rotate x basis vector by angle φ counter-clockwise

$$\begin{pmatrix} 1 & 0 \\ \tan(\phi) & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y + \tan(\phi) \cdot x \end{pmatrix}$$



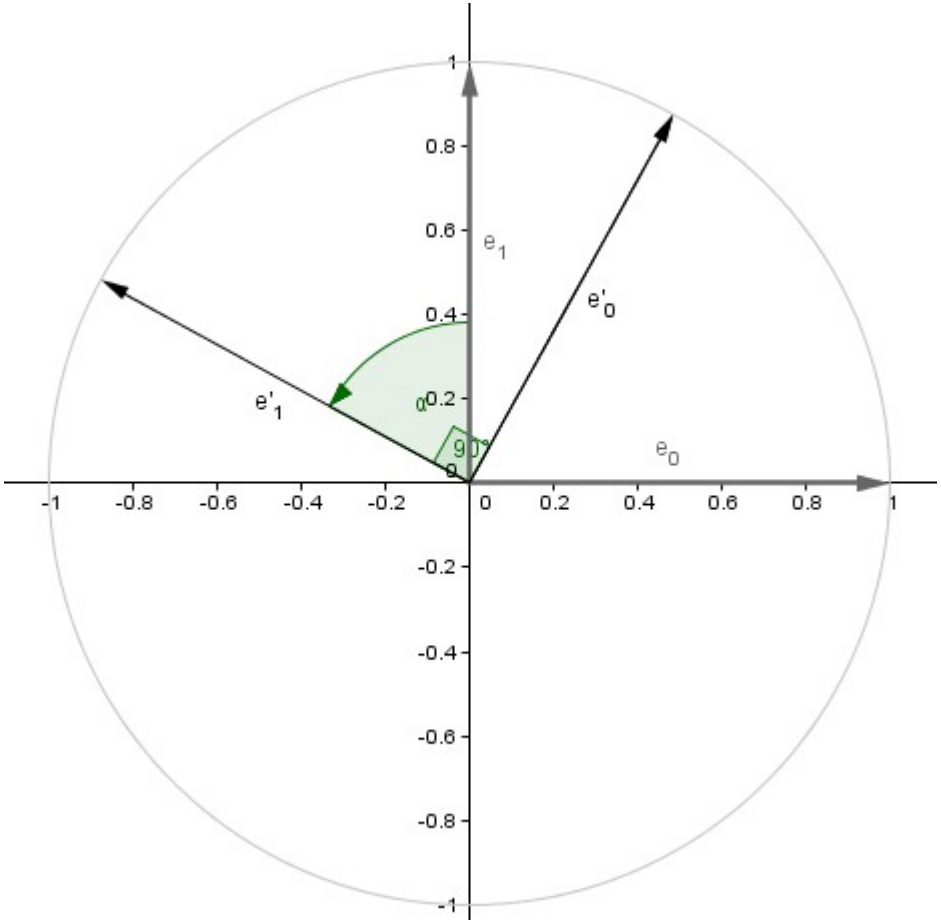
- Shear-y, we rotate y basis vector by angle φ clockwise

$$\begin{pmatrix} 1 & \tan(\phi) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + \tan(\phi) \cdot y \\ y \end{pmatrix}$$



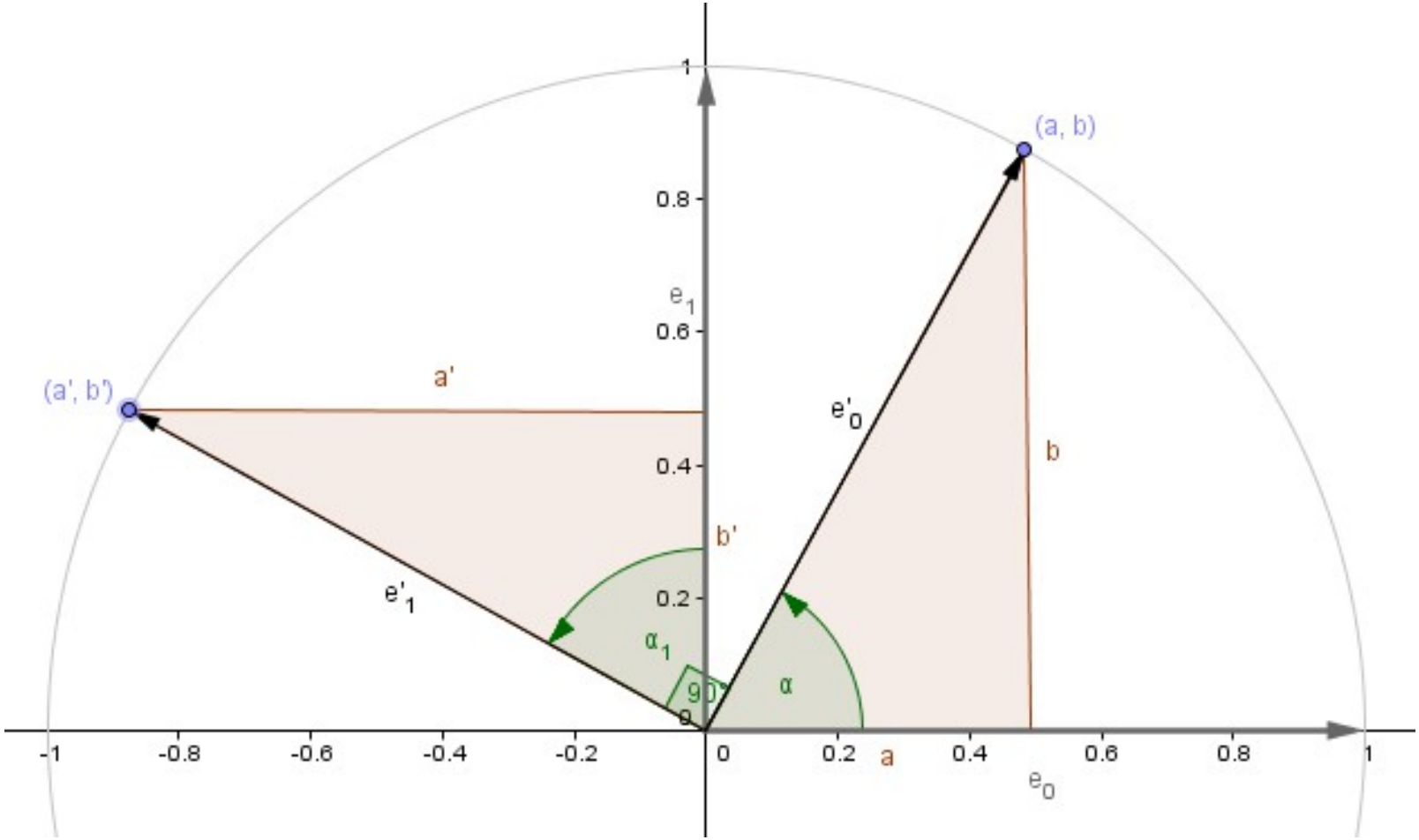
Rotation

- Shearing rotated only one axis
- Also changed the size of the basis vector
- Can we do better?



Did you notice that the columns of the transformation matrix show the coordinates of the new basis vectors?

Rotation



$$e'_0 = (|a|, |b|) = (\cos(\alpha), \sin(\alpha))$$

$$e'_1 = (|a'|, |b'|) = (-\sin(\alpha), \cos(\alpha))$$

$$\cos(\alpha) = \frac{|a|}{|e'_0|} = \frac{|a|}{1} = |a|$$

Rotation

- So if we rotate by α in counter-clockwise order in 2D, the transformation matrix is:

$$\begin{array}{cc} e'_0 & e'_1 \\ \downarrow & \downarrow \\ \end{array}$$

$$\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$$

- In 3D we can do rotations in each plane (xy, xz, yz), so there can be 3 different matrices.

Rotation

- To do a rotation around an arbitrary axis, we can:
  - Rotate that axis to be the x-axis
  - Rotate around the new x-axis
  - Invert the first rotations
    (move the old x-axis back)

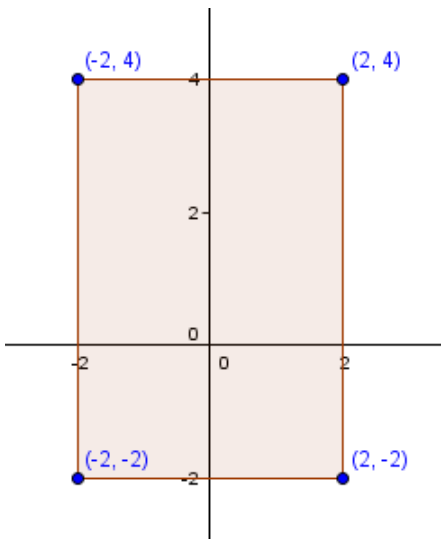
$$\begin{pmatrix} 1 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

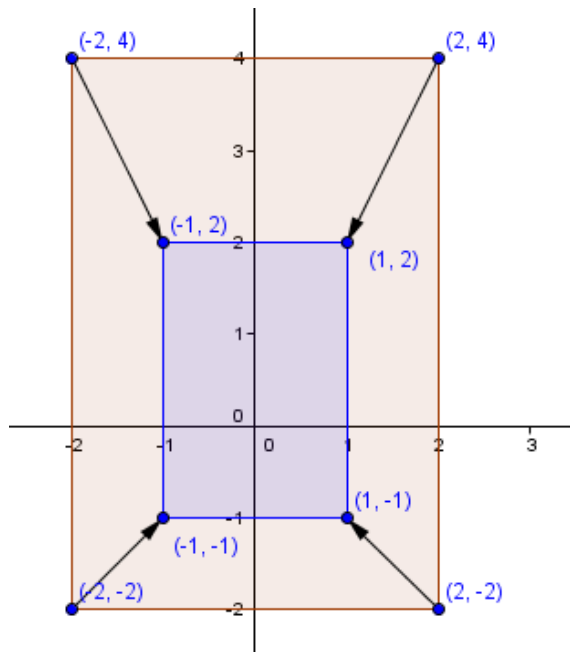
- OpenGL provides a command for rotating around a given axis.
- Sometimes quaternions are used for rotations.

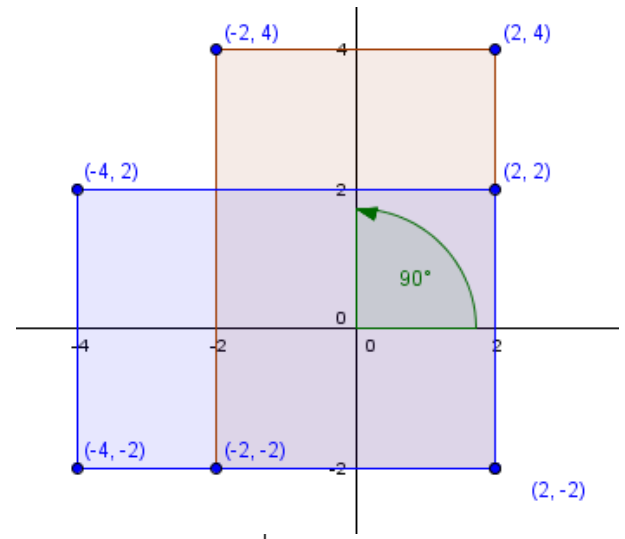
Quaternions are elements of a number system that extend the complex numbers...

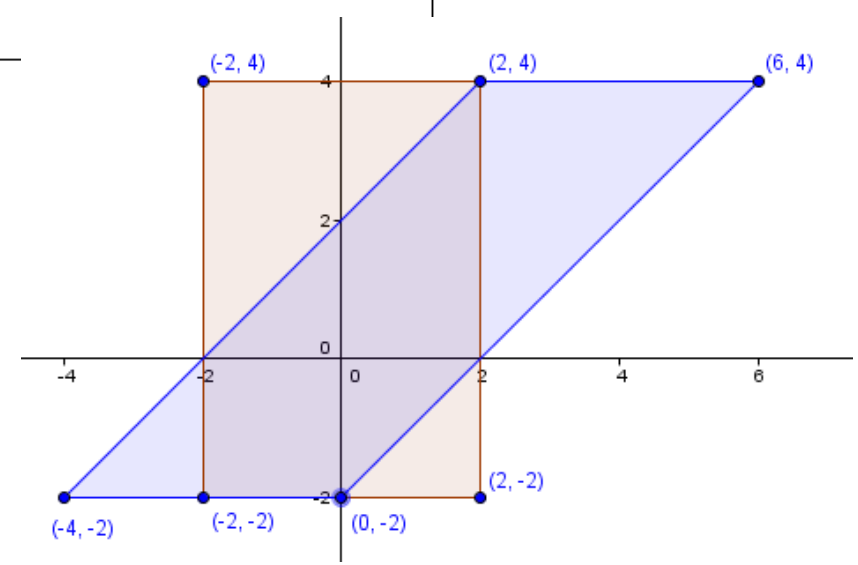
Do we have everything now?

- We can scale, share and rotate our geometry around the origin...





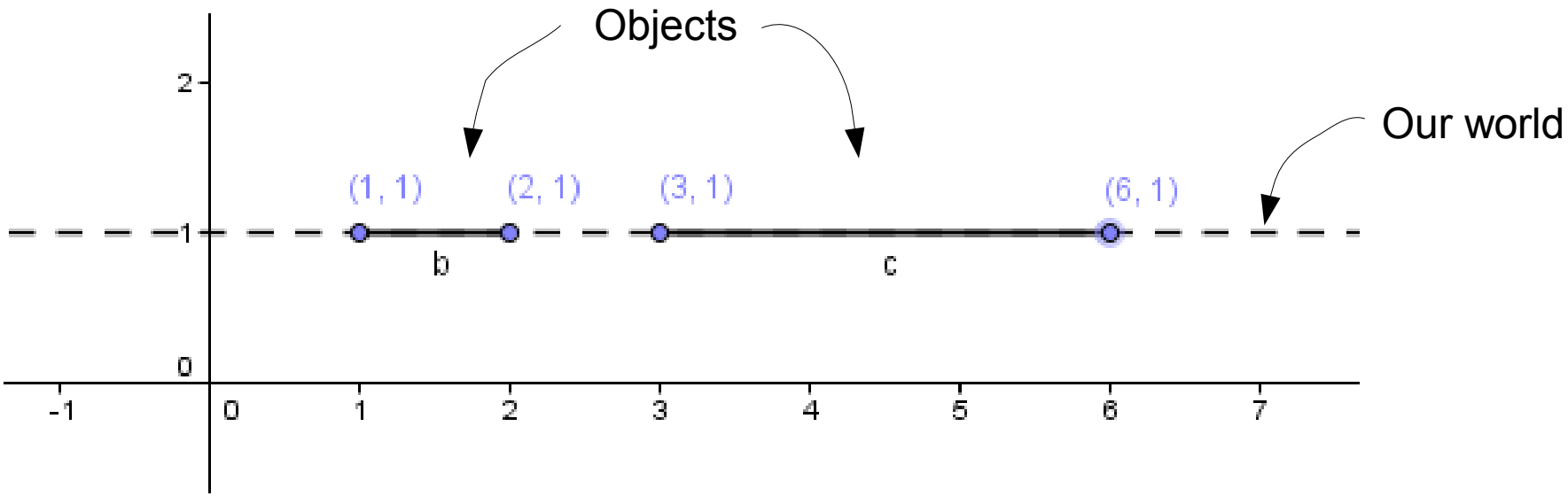




What if we have an object not centered in the origin?

Translation

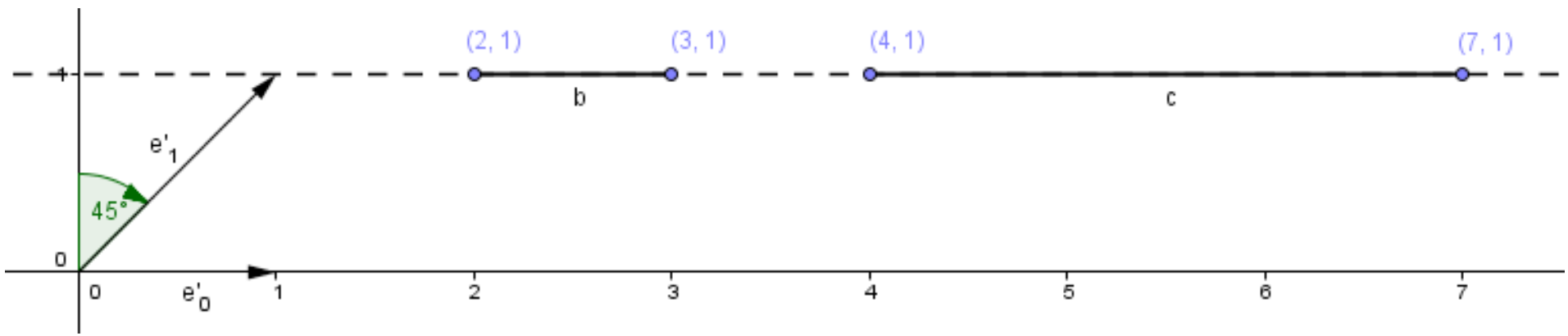
- Imagine that our 1D world is located at y=1 line in 2D space.



- Notice that all the points are in the form: (x, 1)

Translation

- What happens if we do shear-y(45°) operation on the 2D world?



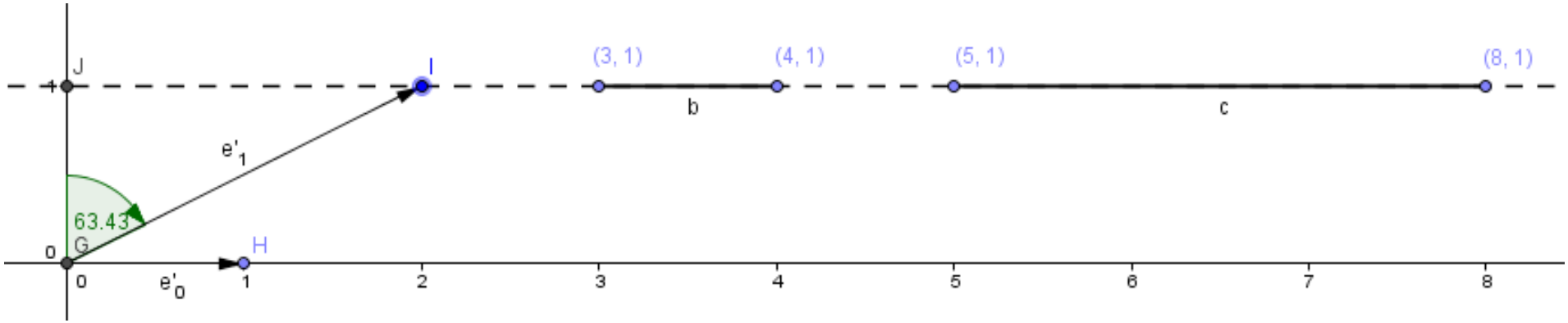
- Everything in our world has moved magically one x-coordinate to the right...

tan(45°) = 1

Translation

- What if we do shear-y(63.4°)?

tan(63.4°) = 2



- Everything has now moved 2 x-coordinates to the right from the original position
- We can do translation / shift!

Translation

- When we represent our points in one dimension higher space, where the extra coordinate is 1, we get to the **homogeneous** space.

$$\begin{pmatrix} 1 & x_t \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ z + z_t \\ 1 \end{pmatrix}$$

Transformations

- This together gives us a very good toolset to transform our geometry as we wish.

Linear transformations

Translation column

Affine trasnformation

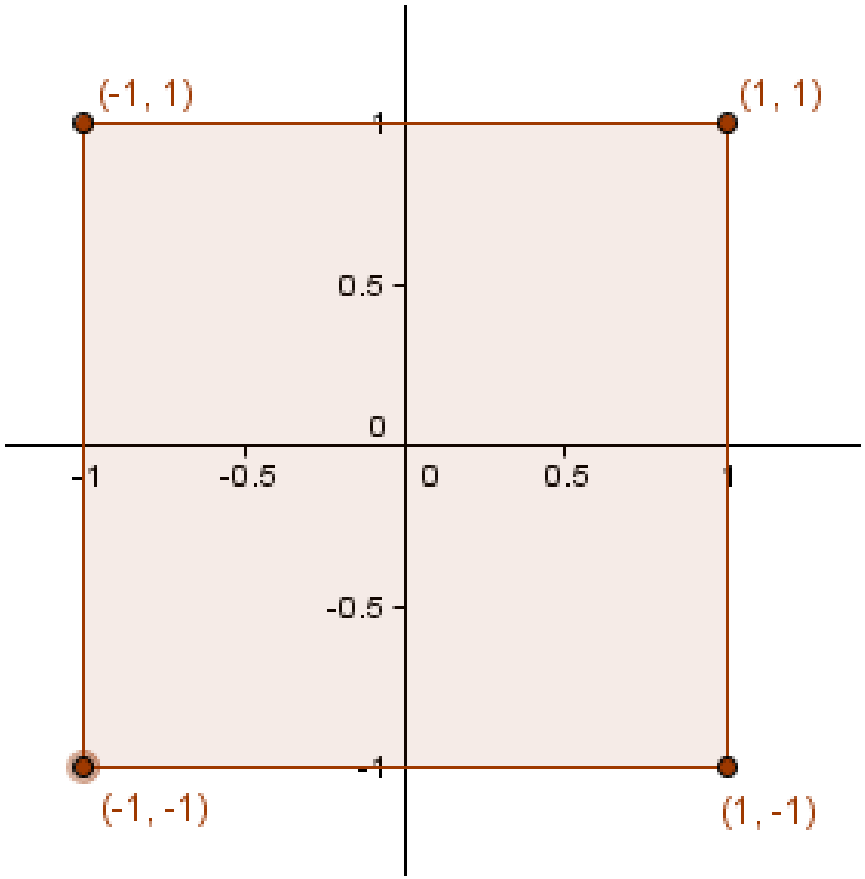
$$\begin{pmatrix} a & b & c & x_t \\ d & e & f & y_t \\ g & h & i & z_t \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} ax+by+cz+x_t \\ dx+ey+fz+y_t \\ gx+hy+iz+z_t \\ 1 \end{pmatrix}$$

Used for perspective projection...

Multiple transformations

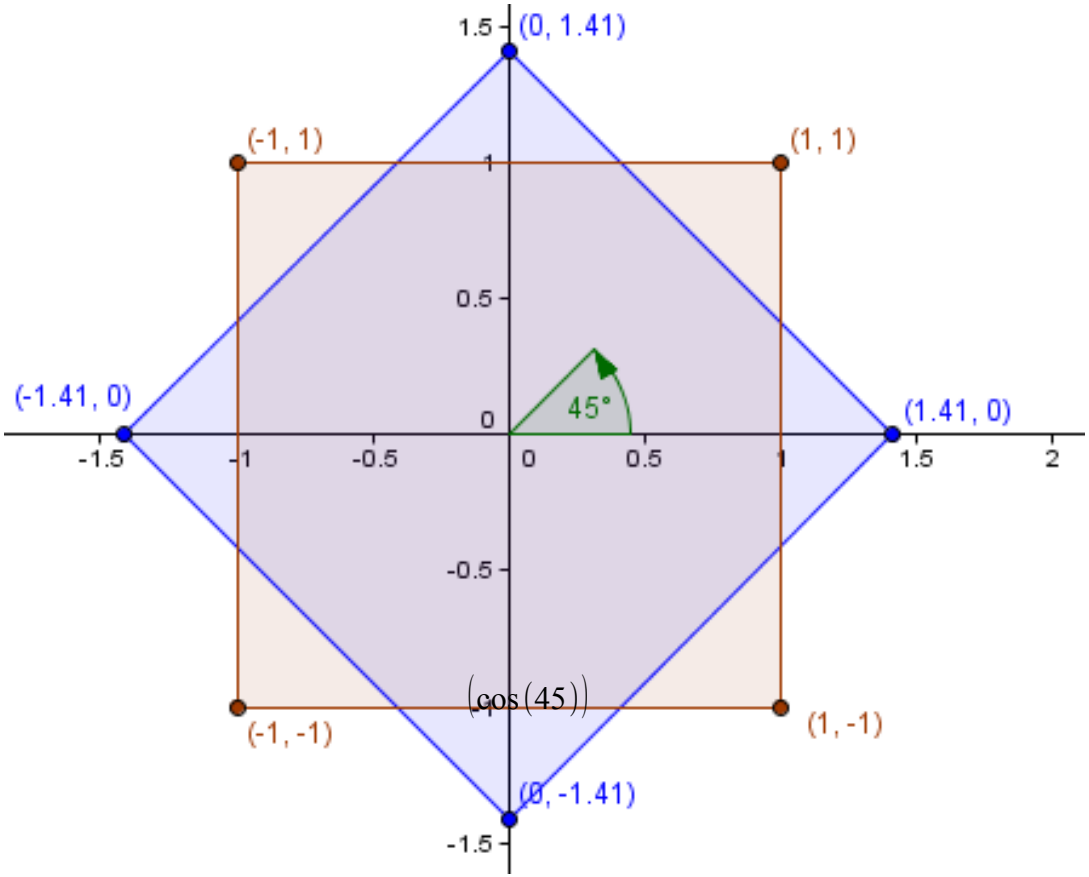
- Everything starts from the origin!
- To apply multiple transformations, just multiply matrices.

Multiple transformations



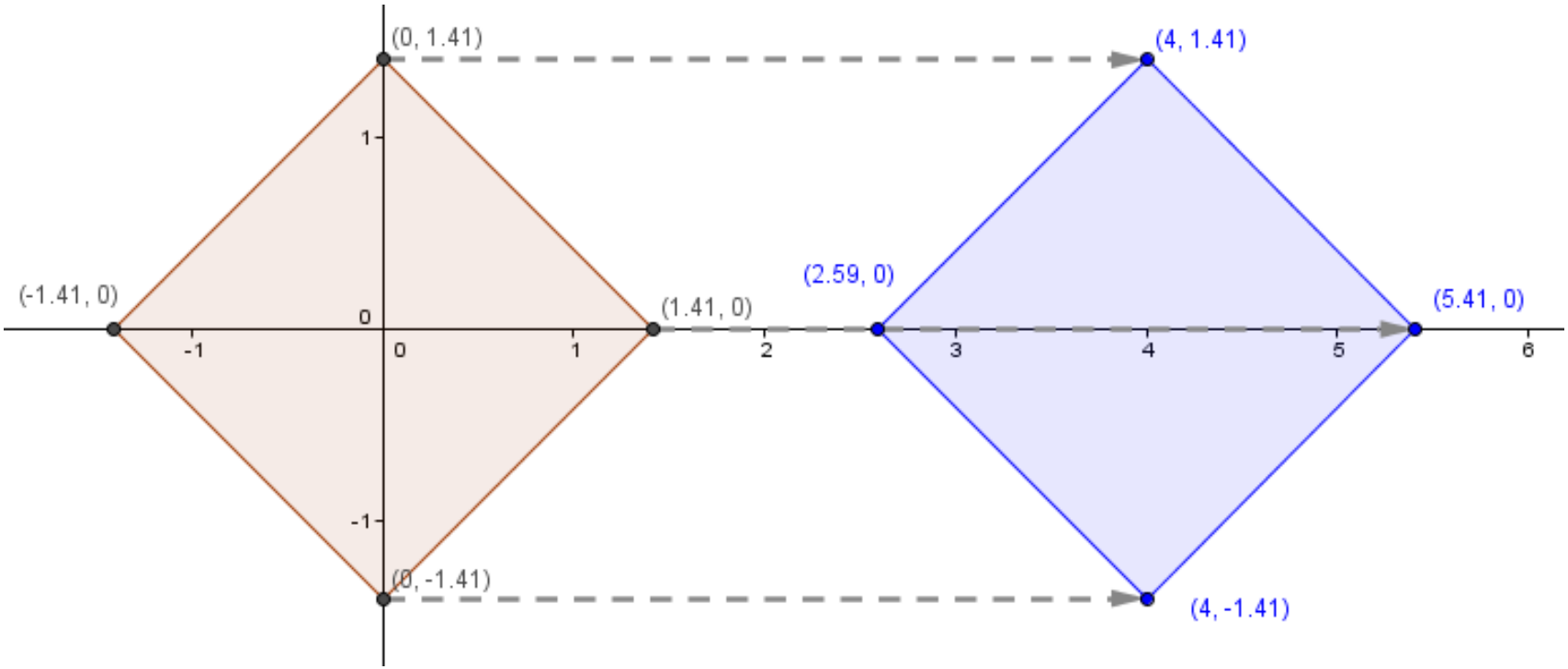
Our initial geometry defined by vertices: (-1, -1), (1, -1), (1, 1), (-1, 1)

Multiple transformations



$$\begin{pmatrix} \cos(45°) & -\sin(45°) & 0 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiple transformations



$$\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiple transformations

- We can combine the transformations to a single matrix.

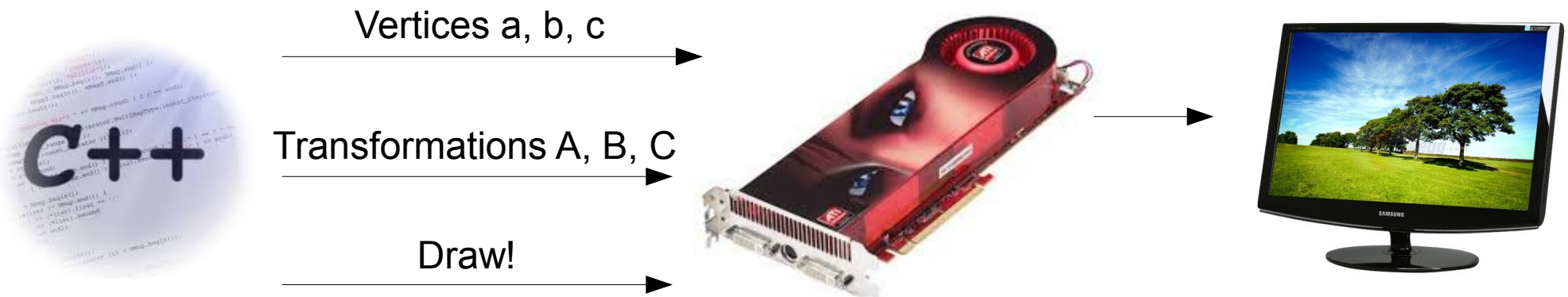
$$\begin{pmatrix} \cos(45°) & -\sin(45°) & 0 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(45°) & -\sin(45°) & 4 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- This also works for combining different linear transformations, but the resulting matrix isn't that clear...
- Order of transformations / matrices is important!
- http://tume-maailm.pri.ee/ylikool/ComputerGraphicsSeminar/demo

Now we know how it's supposed to go...

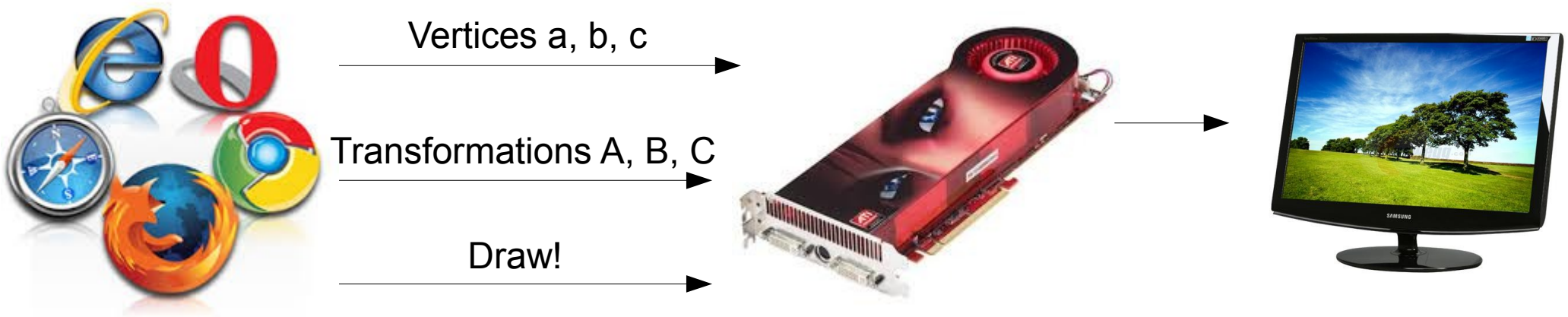
OpenGL

- GPU API
- I.e a set of commands that program can give to GPU
- Supported in many languages



WebGL

- GPU API in JavaScript
- Supported by major browsers
- THREE.js – Higher level library to ease your coding: http://threejs.org/docs/



Forward is backward

- Actually you need to tell GPU commands in this order:
  - Transformations
  - Vertices

```
glTranslatef(0.0, 0.0, -1.0);
glRotatef(60, 1.0, 0.0, 0.0);
glRotatef(-20, 0.0, 0.0, 1.0);

glBegin(GL_QUADS);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
glEnd();
```

Prior to OpenGL 3

State machine

- GPU acts like a state machine

GPU is now in a state to receive quad vertices

Sending vertices to GPU

Stop being in that state

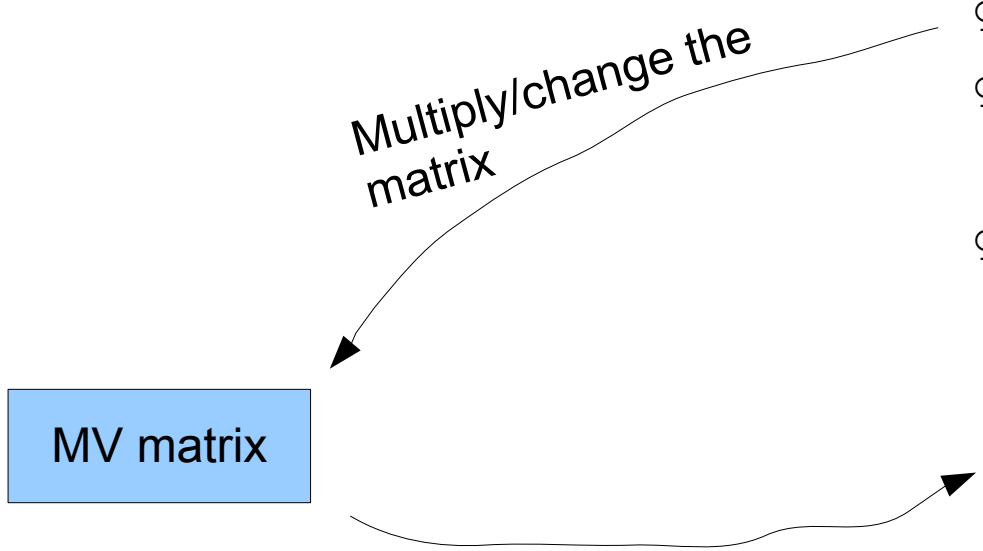
```
glBegin(GL_QUADS);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
glEnd();
```

Prior to OpenGL 3

Multiple objects in the scene?

- Each object has its own geometry
- And its own transformations for that geometry
- OpenGL has a single ModelView matrix

```
glTranslatef(0.0, 0.0, -1.0);
glRotatef(60, 1.0, 0.0, 0.0);
glRotatef(-20, 0.0, 0.0, 1.0);

glBegin(GL_QUADS);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
    glVertex3fv(...);
glEnd();
```

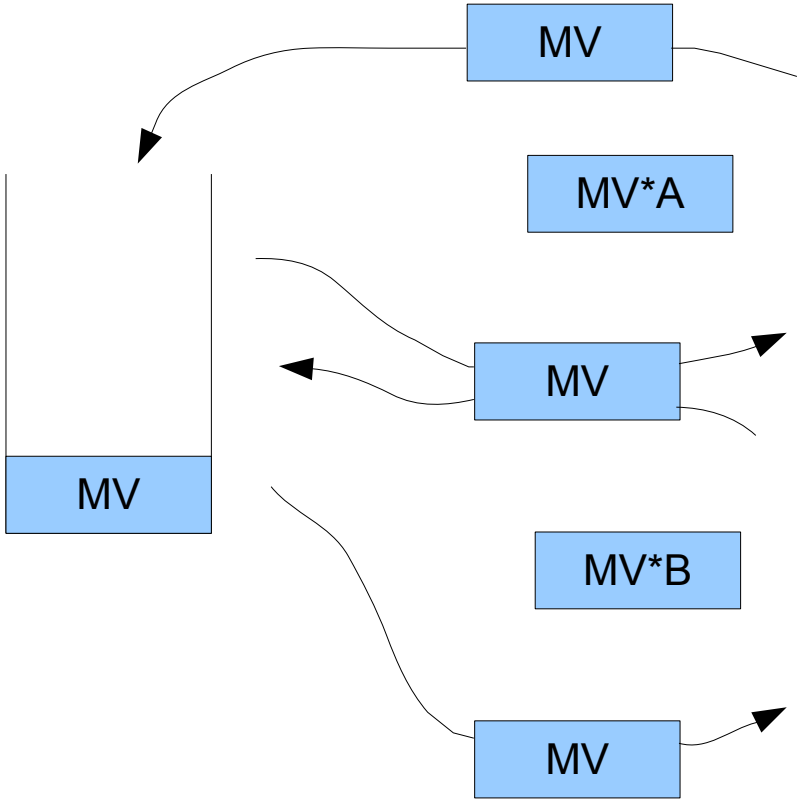


Vertices will be multiplied with the matrix

Prior to OpenGL 3

Multiple objects in the scene

- OpenGL has a matrix stack
- We can push a copy to the stack (save)
- We can pop the top matrix from the stack (load)

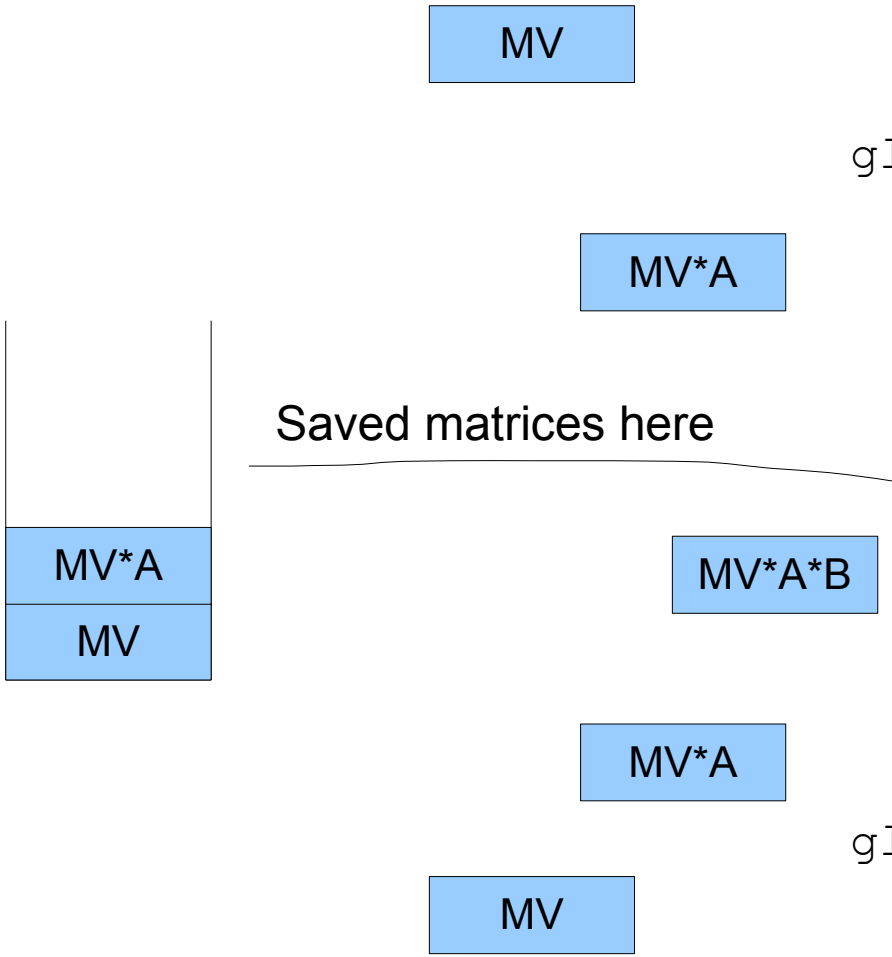



```
glPushMatrix();
    glTranslatef(10.0, 0.0, -1.0);
    drawCube();
glPopMatrix();

glPushMatrix();
    glTranslatef(-10.0, 0.0, -1.0);
    drawSphere();
glPopMatrix();
```

Prior to OpenGL 3

Multiple objects in the scene

- More complex geometry for a single object

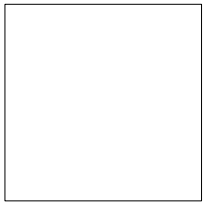
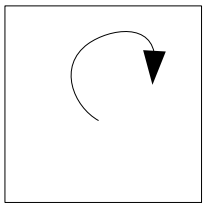
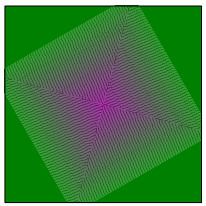



```
glPushMatrix();
    glTranslatef(10.0, 0.0, -1.0);
    drawPalm();

    glPushMatrix();
        glTranslatef(-1.0, 0.0, 0.0);
        drawFingers()
    glPopMatrix();

glPopMatrix();
```

Prior to OpenGL 3

Old and new OpenGL?

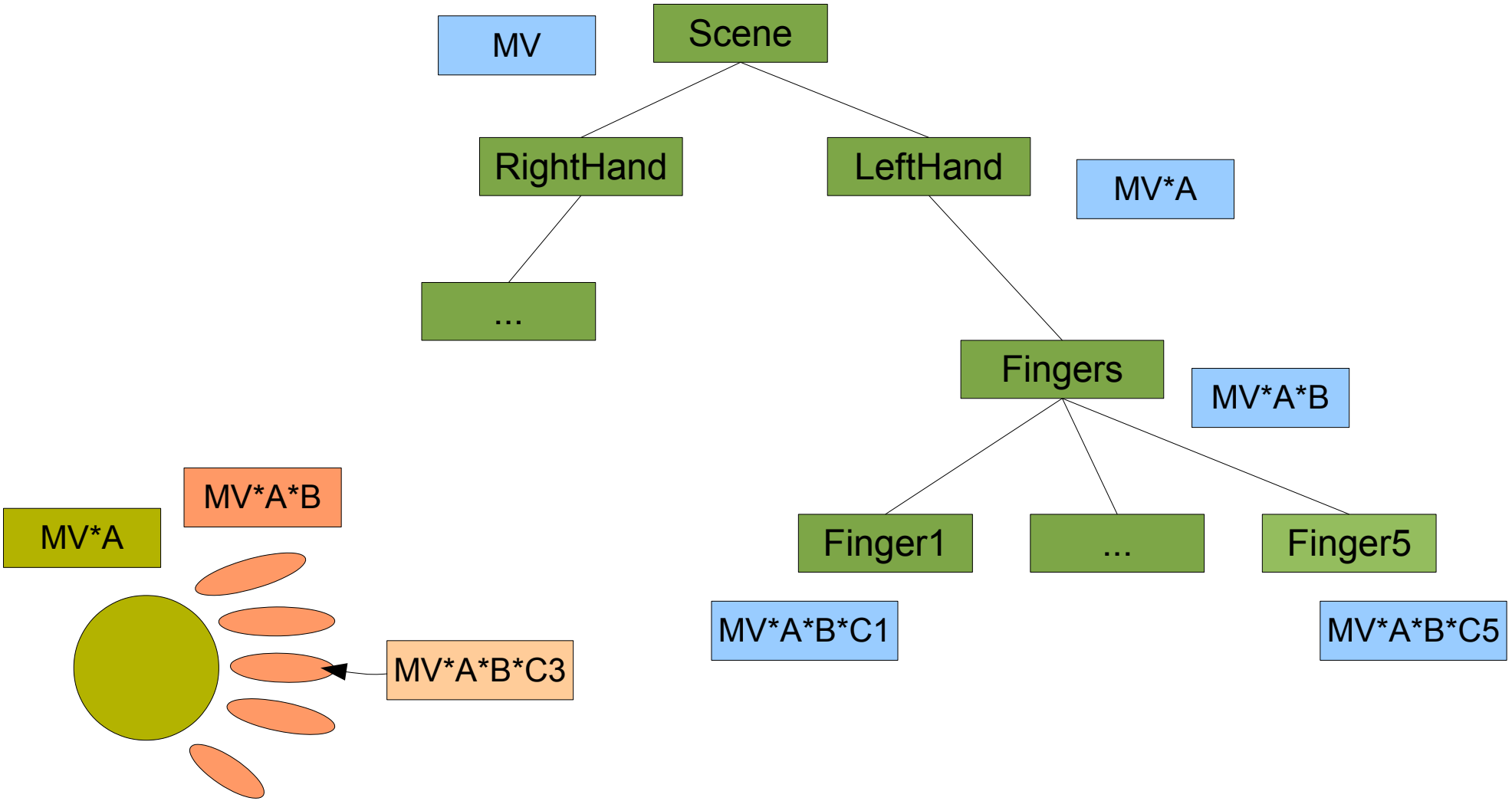
- Lot has changed from OpenGL 3.
- Everything old still works in compatibility mode.

	Prior to OpenGL 3	OpenGL 3+
	•glBegin(...) •glVertex(...) •glEnd(...)	•Vertex Array Object (VAO) •Vertex Buffer Object (VBO)
	•glTranslate(...) •glRotate(...) •glScale(...)	•Use other Matrix library •Send your matrices to shaders
	•glMaterial(...)	•Vertex Buffer Object

Good tutorials: http://antongerdelan.net/opengl/

Multiple objects in the scene

- Useful to think of the scene as a tree



Next time...

- Graphics pipeline in more detail
- How to define color for our geometry?
- Vertex and fragment shaders
- What else could be done?